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KEY

TO

DODD'S ALGEBRA:

CONTAINING

ALL THE PROBLEMS, AND THE MORE DIFFICULT EQUATIONS, IN THAT WORK,

WITH THEIR SOLUTIONS:

DESIGNED TO FACILITATE THE LABOR OF TEACHERS.

BY JAMES WILLIAM DODD, A. B.,
PRINCIPAL OF BETHEL ACADEMY.

NEW YORK:
PRATT, OAKLEY AND COMPANY,
21 MURRAY STREET.
1859.

Educ 7125.54.33c

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J. P. JONES & CO., STEREOTYPERS,
183 WILLIAM STREET.

C. A. ALVORD, PRINTER,
29, & 31 GOLD STREET.

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KEY TO DODD'S ALGEBRA.

PROBLEMS

In Simple Equations of one Unknown Quantity.

1. What number is that to the double of which if 13 be added, the sum will be 75?

Let x represent the number; then
    $2x$ represents *double* the number;
and by the conditions of the problem, the Equation will be

$$2x+13=75.$$

This Equation will give $x=31$.

2. Find a number such that if it be multiplied by 5, and 24 be subtracted from the product, the remainder will be 36.

Let x represent the number; then
    $5x$ represents 5 times the number;
and by the conditions of the problem, the Equation will be

$$5x-24=36;$$

which will give $x=12$.

3. What number is that to $\frac{1}{3}$ of which if 25 be added, the sum obtained will be equal to the number itself *minus* 39?

Let x represent the number; then

$\frac{x}{3}$ represents *one third* of the number;

and the Equation will be

$$\frac{x}{3}+25=x-39;$$

from which we shall find $x=96$.

4. Find a number such that if $\frac{1}{4}$ of it be subtracted from three times the number, the remainder will be 77.

Let x represent the number; then

$\frac{x}{4}$ represents *one* fourth of the number; and

$3x$ represents *three times* the number.

We shall then have the Equation

$$3x-\frac{x}{4}=77;$$

from which the value of x will be found equal to 28.

5. Find what number added to the sum of one half, one third, and one fourth of itself will be equal to 4 added to twice the number.

Let x represent the number; then

$\frac{x}{2}$ represents *one half* of the number;

$\frac{x}{3}$ represents *one third* of the number; and

$\frac{x}{4}$ represents *one fourth* of the number;

and, by the conditions of the problem, the Equation will be

$$\frac{x}{2}+\frac{x}{3}+\frac{x}{4}+x=2x+4;$$

from which we shall find $x=48$.

6. Divide the number 165 into two such parts, that the less may be equal to $\frac{1}{10}$ of the greater.

Let x represent the *less* part; then

$165-x$ represents the *greater;*

and the Equation will be

$$x=\frac{165-x}{10}.$$

This Equation will give $x=15$, the *less* part; hence $165-15=150$ is the *greater*.

7. Divide the number 100 into two such parts that six times the less may be equal to twice the greater.

Let x represent the *less* part; then
$100-x$ represents the greater;
$6x$ represents 6 times the less; and
$200-2x$ represents *twice* the greater.
We shall then have the Equation

$$6x=200-2x;$$

which will give $x=25$, the less part; hence $100-25=75$ is the *greater*.

8. It is required to divide 75 into two such parts that 3 times the greater may exceed 7 times the less by 15.

Let x represent the *less* part; then
$75-x$ represents the *greater;*
$225-3x$ represents 3 times the greater; and
$7x$ represents 7 times the less.
The Equation will then be

$$225-3x=7x+15.$$

This Equation will give $x=21$, the *less* part; hence $75-21=54$ is the *greater*.

9. What sum of money is that to which if \$100 be added, $\frac{2}{3}$ of the amount will be \$400?

Let x represent the sum; then
$\frac{2}{3}(x+100)$, or $\frac{2x+200}{3}$, represents $\frac{2}{3}$ of the *amount* obtained after adding \$100.
The Equation will then be

$$\frac{2x+200}{3}=400;$$

which will give $x=\$500$.

10. A prize of \$100 is to be divided between two persons—the share of the first being $\frac{7}{9}$ of that of the other. What are the shares?

Let x represent the share of the *second;* then
$100-x$ represents the share of the *first;*
and the Equation will be

$$100-x=\frac{7x}{9};$$

from which x will be found $=\$56\frac{1}{4}$, the share of the *second;* hence $100-56\frac{1}{4}=\$43\frac{3}{4}$, the share of the *first.*

11. A post has $\frac{1}{4}$ of its length in the mud, $\frac{1}{3}$ of it in the water, and 15 feet above the water. What is the length of the post?

Let x represent the length, *in feet;* then
$\frac{x}{4}$ represents the part in the *mud;*
$\frac{x}{3}$ represents the part in the *water;* and
15 feet is the part *above* the water.
The Equation will then be

$$\frac{x}{4}+\frac{x}{3}+15=x;$$

from which x will be found $=36$ feet.

12. Find a number such that if it be divided by 12, the divisor, dividend, and quotient together shall make 64.

Let x represent the number; then
12 is the *divisor;*
x represents the *dividend;* and
$\frac{x}{12}$ represents the *quotient.*
We shall then have the Equation

$$12+x+\frac{x}{12}=64.$$

This Equation will give $x=48$.

13. In a mixture of wine and cider, $\frac{1}{2}$ of the whole *plus* 25 gallons was wine, and $\frac{1}{3}$ part *minus* 5 gallons was cider. What was the whole number of gallons in the mixture?

Let x represent the number of gallons; then

$\frac{x}{2}+25$ represents the number of gallons of *wine;* and

$\frac{x}{3}-5$ represents the number of gallons of *cider;*

and the Equation will be

$$\frac{x}{2}+25+\frac{x}{3}-5=x.$$

This Equation will give $x=120$.

14. After a person had expended \$10 more than $\frac{1}{3}$ of his money, he had \$15 more than $\frac{1}{2}$ of it remaining. What sum had he at first?

Let x represent the sum; then

$\frac{x}{3}+10$ represents the amount *expended;* and

$x-\frac{x}{3}-10$ represents the amount *remaining.*

Then, by the conditions of the problem, we shall have

$$x-\frac{x}{3}-10=\frac{x}{2}+15;$$

from which we shall find $x=\$150$.

15. Divide the number 91 into two such parts that if the greater be divided by their difference, the quotient may be 7.

Let x represent the *greater* part; then
$91-x$ represents the *less;* and
$x-(91-x)$, or $2x-91$, represents their *difference;*
and the Equation will be

$$\frac{x}{2x-91}=7.$$

This Equation will give $x=49$ the *greater* part; hence $91-49=42$, is the *less* part.

16. A and B had equal sums of money; the first paid away \$25, and the second \$60, when it appeared that A had twice as much left as B. What sum had each?

Let x represent the sum each had; then
$x-25$ represents the sum A had after spending \$25; and
$x-60$ represents the sum B had after spending \$60.
Then, by the conditions of the problem, we shall have

$$x-25=2(x-60);$$

from which we shall find $x=\$95$.

17. After paying away $\frac{1}{4}$ of my money, and then $\frac{1}{5}$ of what was left, I had \$180. What sum had I at first?

Let x represent the sum; then
$\frac{x}{4}$ represents the first payment;
$\frac{1}{5}$ of $\frac{3x}{4}$, or $\frac{3x}{20}$, represents the second payment; and
$x-\frac{x}{4}-\frac{3x}{20}$, or $\frac{3x}{5}$, is the remainder after *both payments.*

We shall then have the Equation

$$x-\frac{x}{4}-\frac{3x}{20}=180;$$

which will give $x=\$300$.

18. A line 37 feet in length is to be divided into 3 parts, so that the first may be 3 feet less than the second, and the second 5 more than the third. What are the parts?

Let x represent the length of the *third* part; then
$x+5$ represents the length of the 2d part; and
$x+2$ represents the length of the 1st part.
The Equation will then be

$$x+(x+5)+(x+2)=37;$$

from which x will be found $=10$ feet, the *third* part; hence $10+5=15$ feet, the *second* part; and $10+2=12$ feet, the *first* part.

19. A can perform a piece of work in 12 days, and B can perform the same in 15 days. In what time could both together do the work?

Let x represent the number of days.

Then, since A could do $\frac{1}{12}$ of the work, and B $\frac{1}{15}$ of it in 1 day,

$\frac{x}{12}$ represents the part A could do in x days ; and

$\frac{x}{15}$ represents the part B could do in x days.

The Equation will then be

$$\frac{x}{12}+\frac{x}{15}=1, \textit{ the entire work.}$$

This Equation will give $x=6\frac{2}{3}$ days.

20. If A can mow a certain field in 6 days, B in 8 days, and C in 5 days, in what time could the three together do it ?

Let x represent the number of days.

Then, since A could mow $\frac{1}{6}$ of the field, B $\frac{1}{8}$ of it, and C $\frac{1}{5}$ of 1 day,

$\frac{x}{6}$ represents the part A could mow in x days.

$\frac{x}{8}$ represents the part B could mow in x days ; and

$\frac{x}{5}$ represents the part C could mow in x days.

We shall then have the Equation

$$\frac{x}{6}+\frac{x}{8}+\frac{x}{5}=1, \text{ the } \textit{entire field},$$

which will give $x=2\frac{2}{59}$ days.

21. Out of a cask of wine which had leaked away a third part, 20 gallons were afterwards drawn, and the cask was then found to be but half full. How much did it hold ?

Let x represent the number of gallons ; then

$\frac{x}{3}$ represents the number *lost by leakage ;*

$\frac{2x}{3}$ represents the number then *remaining ;* and

$\frac{2x}{3}-20$ represents the number remaining after 20 gallons were drawn.

The Equation will then be

$$\frac{2x}{3}-20=\frac{x}{2};$$

from which we shall find $x=120$ gallons.

22. It is required to divide \$300 between A, B, and C, so that A may have twice as much as B, and C as much as the other two together.

Let x represent B's share; then
$2x$ represents A's share; and
$3x$ represents C's share.
We shall then have the Equation

$$x+2x+3x=300;$$

which will give $x=\$50$, B's share; hence $2\times 50=\$100$ is A's share; and $3\times 50=\$150$ is C's share.

23. A gentleman spends $\frac{2}{3}$ of his yearly income in board and lodging, $\frac{2}{3}$ of the remainder in clothes, and then has \$20 left. What is the amount of his income?

Let x represent his income; then
$\frac{2x}{3}$ represents the amount spent in *board and lodging;*
$\frac{2}{3}$ of $\frac{x}{3}$, or $\frac{2x}{9}$, represents the amount spent in *clothes;* and
$x-\frac{2x}{3}-\frac{2x}{9}$ represents the amount remaining.
We shall then have

$$x-\frac{2x}{3}-\frac{2x}{9}=20;$$

from which x will be found equal to \$180.

24. A person, at the time he was married, was three times as old as his wife, but 15 years afterwards, he was only twice as old. What were their ages on their wedding day?

Let x represent the *wife's* age; then
$3x$ represents the *husband's* age;
$x+15$ represents the wife's age 15 years afterwards; and
$3x+15$ represents the husband's age 15 years afterwards.
Then, by the conditions of the problem, we shall have

$$3x+15=2(x+15);$$

from which x will be found $=15$ years, the wife's age; hence $3\times15=45$ years, was the husband's age.

25. Two persons, A and B, lay out equal sums of money in trade; the first gains \$126, and the second loses \$87, and A's money is now double of B's. What did each lay out?

Let x represent the sum each laid out; then
$x+126$ represents the sum A had after gaining \$126; and
$x-87$ represents the sum B had after losing \$87.
Then the Equation will be,

$$x+126=2(x-87);$$

which will give $x=\$300$.

26. A courier who travels 60 miles a day had been despatched 5 days, when a second is sent to overtake him, who goes 75 miles a day. In what time will he overtake him?

Let x represent the number of days; then
$x+5$ represents the number of *days* the *first* traveled;
$60(x+5)$ represents the number of *miles* the *first* traveled; and
$75x$ represents the number of *miles* the *second* traveled.
Since both traveled the same number of miles, we shall have

$$60(x+5)=75x;$$

from which we shall find $x=20$ days.

27. An island is 60 miles in circumference, and two persons, A and B, start together to travel the same way around it; A goes 15 miles a day, and B 20. In what time would the two come together again?

Let x represent the number of days; then
$15x$ represents the distance A traveled; and
$20x$ represents the distance B traveled.
Since the distance B traveled exceeds the distance A traveled, by the circumference of the island, the Equation will be

$$20x-15x=60;$$

from which x will be found equal to 12 days.

28. A man and his wife usually drank out a cask of beer in 12 days, but when the man was from home it lasted the woman 30 days. How many days would the man alone be in drinking it?

Let x represent the number of days; then

$\frac{1}{x}$ represents the part he would drink in one day; and

$\frac{1}{30}$ is the part the *woman* would drink in one day.

We shall then have the Equation

$$\frac{1}{x}+\frac{1}{30}=\frac{1}{12},\ \textit{the part both would drink in } 1 \textit{ day.}$$

From this Equation we shall find $x=20$ days.

29. If A and B together can do a piece of work in 9 days, and A alone could do it in 15 days, in what time could B alone do the work?

Let x represent the number of days; then

$\frac{1}{x}$ represents the part B could do in 1 day; and

$\frac{1}{15}$ is the part A could do in 1 day.

We shall then have the Equation

$$\frac{1}{x}+\frac{1}{15}=\frac{1}{9},\ \textit{the part both could do} \text{ in 1 day.}$$

This Equation will give $x=22\frac{1}{2}$ days.

30. The hour and minute hands of a watch or clock are exactly together at 12 o'clock. When are they next together?

The dial face of the clock is divided into 60 *equal minute spaces.*

Let x represent the number of these spaces passed over by the *hour* hand before the two hands are next together.

Then, since the *minute* hand moves 12 times as fast as the *hour* hand,

$12x$ represents the number of spaces passed over by the *minute* hand before the two are next together.

But $60+x$ also represents the number of these spaces passed over by the minute hand in the *same* time.

We shall then have the Equation

$$12x=60+x;$$

which will give $x=5\frac{5}{11}$ spaces; hence they will be together $5\frac{5}{11}$ minutes past 1 o'clock.

31. It is required to divide \$1000 between A, B, and C, so that A shall have half as much as B, and C half as much as A and B together.

Let x represent A's share; then
$2x$ represents B's share; and
$\frac{3x}{2}$ represents C's share.
The Equation will then be

$$x+2x+\frac{3x}{2}=1000;$$

which will give $x=\$222\frac{2}{9}$, A's share; then $2\times222\frac{2}{9}=\$444\frac{4}{9}$, B's share; and $\frac{3}{2}\times222\frac{2}{9}=\$333\frac{1}{3}$, C's share.

32. A person being asked the hour, answered that the time past noon was $\frac{2}{3}$ of the time till midnight. What was the hour?

Let x represent the number of *hours* past noon; then
$12-x$ represents the number of hours till midnight; and, by the conditions of the problem, we have the Equation

$$x=\frac{2}{3}(12-x);$$

which will give $x=4\frac{4}{5}$ hours $=4h.\ 48m.$

33. It is required to divide the number 60 into two such parts, that their product shall be equal to the square of the less. What are the parts?

Let x represent the *less* part; then
$60-x$ represents the *greater* part;
$60x-x^2$ represents their *product*; and
$3x^2$ represents *three times* the *square* of the less.
The Equation will then be

$$60x-x^2=3x^2.$$

Dividing both sides by x, we have

$$60-x=3x;$$

from which x will be found $=15$, the *less* part; hence $60-15=45$, the *greater* part.

34. How much wine at 90 cents a gallon, and how much at \$1.50 a gallon, will be required to form a mixture of 20 gallons which shall be worth \$1.25 a gallon?

Let x represent the number of gallons at 90 cents a gallon; then
$20-x$ represents the number of gallons at \$1.50 a gallon;
$90x$ represents the *value* of the 1st kind in *cents;*
$150(20-x)$ represents the *value* of the 2d kind in *cents;* and
20×125, or 2500 cents, is the value of the *mixture.*

The Equation, then, will evidently be

$$90x+150(20-x)=2500;$$

which will give $x=8\frac{1}{3}$ gallons, the quantity at 90 cents a gallon; hence $20-8\frac{1}{3}=11\frac{2}{3}$ gallons, the quantity at \$1.50 a gallon.

35. A cistern is supplied with water by three pipes which would severally fill it in 4, 5, and 6 hours. In what time would the three pipes running together fill the cistern?

Let x represent the number of hours.

Then, since the first would fill $\frac{1}{4}$ of the cistern in 1 hour, the second $\frac{1}{5}$ of it, and the third $\frac{1}{6}$ of it,

$\frac{x}{4}$ represents the part the 1st would fill in x hours;
$\frac{x}{5}$ represents the part the 2d would fill in x hours; and
$\frac{x}{6}$ represents the part the 3d would fill in x hours.

Then, we shall have the Equation

$$\frac{x}{4}+\frac{x}{5}+\frac{x}{6}=1, \textit{ the entire cistern,}$$

from which x will be found $=1\frac{23}{37}$ hours.

36. If \$1000 be divided between A, B, and C, so that B shall have as much as A and half as much more, and C as much as B and half as much more, what will be the portion of each?

Let x represent A's share; then

$x+\frac{x}{2}$, or $\frac{3x}{2}$, represents B's share; and

$\frac{3x}{2} + \frac{1}{2}$ of $\frac{3x}{2}$, or $\frac{9x}{4}$, represents C's share.

The Equation will then be

$$x + \frac{3x}{2} + \frac{9x}{4} = 1000;$$

from which x will be found equal to \$$210\frac{10}{19}$, A's share; hence $\frac{3}{2}$ of $210\frac{10}{19}$ = \$$315\frac{15}{19}$, B's share; and $\frac{9}{4}$ of $210\frac{10}{19}$ = \$$473\frac{13}{19}$, C's share.

37. A person has a lease for 99 years, and $\frac{2}{3}$ of the time which has expired on it is equal to $\frac{4}{5}$ of that which remains. Required the time which remains on the lease.

Let x represent the time *remaining;* then
$99 - x$ represents the time *expired.*
Then, by the conditions of the problem, we shall have

$$\frac{2}{3}(99 - x) = \frac{4x}{5};$$

which will give $x = 45$ years.

38. A merchant bought cloth at the rate of \$7 for 5 yards, which he sold again at the rate of \$11 for 7 yards, and gained \$100. How many yards were thus bought and sold?

Let x represent the number of yards.

Then, as $\frac{7}{5}$ of a dollar is what he *paid per yard,* and $\frac{11}{7}$ of a dollar what he *received per yard,*

$\frac{7x}{5}$ represents what he *paid* for x yards; and

$\frac{11x}{7}$ represents what he *received* for x yards.

Since he gained 100 dollars, we shall have the Equation

$$\frac{11x}{7} = \frac{7x}{5} + 100;$$

from which we shall find $x = 583\frac{1}{3}$ yards.

39. A and B together possess the sum of \$9800; and five-sixths of the sum owned by A is the same as four-fifths of the sum owned by B. What is the sum owned by each?

Let x represent the sum owned by A; then
$9800 - x$ represents the sum owned by B.

By the conditions of the problem, we shall have

$$\frac{5x}{6} = \frac{4}{5}(9800-x);$$

which will give $x=\$4800$, A's sum of money; hence $9800-4800=\$5000$, is B's sum of money.

40. The assets of a bankrupt, amounting to $5600, are to be divided among his creditors, A, B, and C, according to their respective claims. A's claim is $\frac{1}{2}$ of B's and C's is $\frac{2}{3}$ of B's. What sum must each of the creditors receive?

Let x represent B's claim; then

$\frac{x}{2}$ represents A's claim; and

$\frac{2x}{3}$ represents C's claim.

The Equation will then be $x+\frac{x}{2}+\frac{2x}{3}=5600$;

which will give $x=\$2584\frac{8}{13}$, B's claim; then $2584\frac{8}{13}\div 2=\$1294\frac{4}{13}$, is A's claim; and $\frac{2}{3}$ of $2584\frac{8}{13}=\$1723\frac{1}{13}$, is C's claim.

PROBLEMS

In Simple Equations of one, two, &c., unknown Quantities.

(85)

1. Find two numbers such, that $\frac{1}{2}$ of the first with $\frac{1}{3}$ of the second shall be equal to 9, and $\frac{1}{4}$ of the first with $\frac{1}{5}$ of the second shall be equal to 5.

Let x represent the *first* and y the *second* number; then, by the conditions of the problem, we shall have

$$\frac{x}{2}+\frac{y}{3}=9; \text{ and}$$

$$\frac{x}{4}+\frac{y}{5}=5.$$

These Equations will give $x=8$, and $y=15$.

2. Divide the number 100 into two such parts that $\frac{1}{3}$ of the first and $\frac{1}{4}$ of the second part shall together make 30.

Let x represent the *first*, and y the *second* part; then we shall have the Equations,

$$x + y = 100; \text{ and}$$

$$\frac{x}{3} + \frac{y}{4} = 30.$$

From these Equations we shall find $x=60$, and $y=40$.

3. Find two numbers such that their sum shall be 60, and the less number $\frac{1}{3}$ of the greater.

Let x represent the *greater*, and y the *less*.
The Equations will then be

$$x+y=60; \text{ and}$$

$$y=\frac{x}{3};$$

which will give $x=45$, and $y=15$.

4. At a certain election, 946 men voted for two candidates, and the successful one had a majority of 558. How many votes were given for each candidate?

Let x represent the number for the *successful*, and y the number for the *unsuccessful* candidate.
We shall have the Equations

$$x+y=946; \text{ and}$$

$$x-y=558;$$

from which Equations x will be found $=752$, and $y=194$.

5. Divide the number 48 into two such parts, that the quotient of the greater part divided by 4 may be equal to 4 times the quotient of the greater part divided by the less.

Let x represent the *greater*, and y the *less* part.
We shall then have the Equations

$$x+y=48; \text{ and}$$

$$\frac{x}{4} = \frac{4x}{y};$$

from which we shall find $x=32$, and $y=16$.

6. A, B, and C, make a joint contribution, which, in the whole,

amounts to \$400; B contributed twice as much as A and \$20 more, and C as much as the other two together. What sum did each contribute?

Let x represent A's contribution, y B's, and z C's.
By the conditions of the problem, the Equations will then be

$$x+y+z=400;$$
$$y=2x+20;\text{ and}$$
$$z=x+y.$$

These three Equations will give $x=60$, $y=140$, and $z=200$.

7. Find three numbers such that the sum of the first and second shall be 35, the sum of the first and third 40, and the sum of the second and third 45.

Let x represent the *first*, y the *second*, and z the third; then the Equations will be

$$x+y=35;$$
$$x+z=40;\text{ and}$$
$$y+z=45;$$

from which we shall find $x=15$, $y=20$, and $z=25$.

8. A sum of money was divided between A and B, so that B's share was $\frac{3}{5}$ of A's, and A's share exceeded $\frac{5}{9}$ of the whole sum by \$50 What was the share of each?

Let x represent A's share, and y B's share; then
$x+y$ represents the *whole sum;* and
$\frac{5x+5y}{9}$ represents $\frac{5}{9}$ of the *whole* sum.

By the conditions of the problem, we shall then have

$$y=\frac{3x}{5};\text{ and}$$

$$x=\frac{5x+5y}{9}+50.$$

These Equations will give $x=\$450$, and $y=\$270$.

9. The stock of three traders amounted to \$760. The shares of the 1st and 2d together exceeded the share of the 3d by \$240; and the share of the 1st was \$360 less than the sum of the shares of the other two. What was the share of each?

Let x represent the share of the *first*, y the share of the *second*, and z the share of the *third;* then we shall have the Equations

$$x+y+z=760\,;$$

$$x+y=z+240\,; \text{ and}$$

$$x=y+z-360.$$

These Equations will give $x=\$200$, $y=\$300$, and $z=\$260$.

10. A man being asked the age of himself and son replied, "If I were $\frac{1}{4}$ as old as I am + 3 times the age of my son I should be 45; and if he were $\frac{1}{4}$ his present age + three times mine, he would be 111." Required their ages.

Let x represent the *father's* age, and y the *son's* age.
Then, by the conditions of the problem, we shall have

$$\frac{x}{4}+3y=45\,; \text{ and}$$

$$\frac{y}{4}+3x=111\,;$$

from which x will be found $=36$, and $y=12$ years.

11. A and B together have \$340, B and C together \$384, and A and C together \$356. What sum has each?

Let x represent A's, y B's, and z C's, amount of money.
We shall then have the Equations

$$x+y=340\,;$$
$$y+z=384\,; \text{ and}$$
$$x+z=356.$$

These three Equations will give $x=\$156$, $y=\$184$, and $z=\$200$.

12. A number which is expressed by two digits is equal to 4 times the sum of its digits, and if 18 be added to the number, its digits will be interchanged with each other. What is the number?

Let x represent the *tens'* and y the *units'* figure of the number.
Then, since the number is equal to ten times the tens' figure + the *units'* figure,

$10x+y$ represents the *required number;* and

$10y+x$ represents the number with its digits *interchanged with each other.*

By the conditions of the problem, the Equations will then be

$$10x+y=4(x+y); \text{ and}$$

$$10x+y+18=10y+x;$$

from which we shall find $x=2$, the *tens'* figure, and $y=4$, the *units'* figure. Hence the number is 24.

13. It is required to divide the number 36 into three such parts, that $\frac{1}{2}$ of the first, $\frac{1}{3}$ of the second, and $\frac{1}{4}$ of the third, shall all be equal to each other. What are the parts?

Let x, y, and z represent the *first*, *second*, and *third* parts, respectively.

The Equations will then be

$$x+y+z=36;$$

$$\frac{x}{2}=\frac{y}{3}; \text{ and}$$

$$\frac{y}{3}=\frac{z}{4}.$$

From these Equations we shall find $x=8$, $y=12$, and $z=16$.

14. A and B have both the same income; A saves $\frac{1}{5}$ of his annually, but B, by spending $50 per annum more than A, at the end of 4 years, finds himself $100 in debt. What is their income?

Let x represent their income; then

$\frac{x}{5}$ represents what A *saves* annually; and

$\frac{4x}{5}$ represents what A *spends* annually; hence

$\frac{4x}{5}+50$ represents what B spends annually.

Now, since B at the end of 4 years is $100 in debt, his *expenses* for that time, amount to 100 dollars more than his income for the *same time*.

Then, since $4x$ represents B's income for 4 years, we shall have

$$4\left(\frac{4x}{5}+50\right)=4x+100;$$

which will give $x=\$125$.

15 A gentleman purchased a chaise, horse, and harness, for $180; the horse cost twice as much as the harness, and the chaise twice as much as the horse and harness together. What was the price of each?

Let x represent the price of the *harness;* then
$2x$ represents the price of the *horse;* and
$6x$ represents the price of the *chaise.*
We shall then have the Equation

$$x+2x+6x=180;$$

which will give $x=\$20$, the price of the *harness;* hence $2\times 20=\$40$, is the price of the horse; and $6\times 20=\$120$, is the price of the chaise.

16. A farmer purchased 100 acres of land for $2450; for a part of the land he paid $20 an acre, and for the other part $30 an acre. How many acres were there in each part?

Let x represent the number of acres in the *first*, and y the number in the *second* part; then
$20x$ represents the cost of the 1st part; and
$30y$ represents the cost of the 2d part.
Hence we shall have the Equations

$$x+y=100; \text{ and}$$

$$20x+30y=2450;$$

which will give $x=55$, and $y=45$ acres.

17. What fraction is that to the numerator of which if 1 be added, the value will be $\frac{1}{2}$; but if 1 be added to the denominator, the value of the fraction will be $\frac{1}{3}$?

Let $\frac{x}{y}$ represent the fraction.

Then, by the conditions of the problem, we shall have

$$\frac{x+1}{y}=\frac{1}{2}; \text{ and}$$

$$\frac{x}{y+1}=\frac{1}{3};$$

which will give $x=3$, and $y=8$. Hence the fraction is $\frac{3}{8}$.

18. A and B together possess an income of $570; if A's income were three times, and B's five times as much as each really is, they would together have $2350. What is the income of each?

Let x represent A's income, and y B's income.
Then the Equations will be

$$+y=570\text{; and}$$
$$3x+5y=2350.$$

These Equations will give $x=\$250$, and $y=\$320$.

19. "How old are we?" said a person to his father: "6 years ago," replied the latter, "I was a third more than 3 times as old as you were; and in three years, if I multiply your age by $2\frac{1}{6}$, it will then be equal to mine." What were their ages?

Let x represent the *father's* age, and y the *son's* age; then
$x-6$ represents the father's age 6 years ago;
$y-6$ represents the son's age 6 years ago;
$x+3$ will be the father's age 3 years hence; and
$y+3$ will be the son's age 3 years hence.
Then, by the conditions of the problem, we shall have

$$x-6=3(y-6)+\frac{y-6}{3}\text{; and}$$
$$x+3=2\tfrac{1}{6}(y+3);$$

from which we shall find $x=36$, and $y=15$ years.

20. Find a number such that if we subtract it from 4980, divide the remainder by 8, and subtract 123 from the quotient, we shall find a remainder equal to the number itself.

Let x represent the number; then
$4980-x$ represents the *remainder* after *subtracting* the number from 4980; and
$\frac{4980-x}{8}$ represents the *quotient* after dividing the remainder by 8.
Then, subtracting 123 from this quotient, we shall have the Equation

$$\frac{4980-x}{8}-123=x;$$

which will give $x=444$.

21. A laborer engaged for 40 days upon these conditions; that for every day he worked he should receive 80 cents, but for every day he was idle he should forfeit 32 cents. At the end of the time he was entitled to \$15.20. How many days did he work, and how many was he idle?

Let x represent the number of days he *worked*, and y the number he *was idle;* then

$80x$ represents the number of cents he *earned;* and
$32y$ represents the number of cents he *forfeited;* also
\$15.20 is equal to 1520 *cents.*

The Equations will then be

$$x+y=40; \text{ and}$$

$$80x-32y=1520.$$

These Equations will give $x=25$, and $y=15$ days.

22. A cistern containing 820 gallons is filled in 20 minutes by 3 pipes, the first of which conveys 10 gallons more, and the second 5 gallons less than the third, per minute. How much flows through each pipe in a minute?

Let x, y, and z respectively represent the number of gallons that flow through each pipe per minute.

Then, since $20x$, $20y$, and $20z$ respectively represent the number of gallons that flow through each pipe in 20 minutes, we shall have the Equation

$$20x+20y+20z=820.$$

And the other Equations will evidently be

$$x=z+10; \text{ and}$$

$$y=z-5.$$

From these three Equations we shall find $x=22$, $y=7$, and $z=12$ gallons.

23. A trader maintained himself for 3 years at an expense of £50 a year, and each year augmented that part of his stock which was not thus expended by $\frac{1}{3}$ thereof. At the end of the third year his original stock was doubled. What was that stock?

Let x represent his stock; then

$x-50$ is what was left after the 1st *expenditure;*

$\left(x-50+\dfrac{x-50}{3}\right)$, or $\dfrac{4x-200}{3}$, is what he had after *augmenting* his stock the 1st time;

$\left(\dfrac{4x-200}{3}-50\right)$, or $\dfrac{4x-350}{3}$, is what was left after the 2d *expenditure;*

$\left(\frac{4x-350}{3}, +\frac{1}{3} \text{ of } \frac{4x-350}{3}\right)$, or $\frac{16x-1400}{9}$, is what he had after *augmenting* his stock the 2d time;

$\left(\frac{16x-1400}{9} -50\right)$, or $\frac{16x-1850}{9}$, is what was left after the 3d *expenditure;* and

$\left(\frac{16x-1850}{9} +\frac{1}{3} \text{ of } \frac{16x-1850}{9}\right)$, or $\frac{64x-7400}{27}$, is what he had after *augmenting* his stock the 3d time.

Then, by the problem, the Equation will be

$$\frac{64x-7400}{27}=2x,$$

which will give $x=$ £740.

24. A and B began to trade with equal sums of money. The first year A gained \$40; and B lost \$40; the second year A lost $\frac{1}{3}$ of what he had at the end of the first, and B gained \$40 less than twice what A lost; when it appeared that B had twice as much money as A. What sum did each begin with?

Let x represent the sum; then

$x+40$ is what A had at the end of the 1st year;

$x-40$ is what B had at the end of the 1st year;

$\left(x+40-\frac{x+40}{3}\right)$, or $\frac{2x+80}{3}$, is what A had at the end of the 2d year.

And since A lost $\frac{x+40}{3}$, and B gained \$40 less than *twice* that sum,

$2\times\frac{x+40}{3}-40$ represents what B gained the 2d year; and

$(x-40+2\times\frac{x+40}{3}-40)$, or $\frac{5x-160}{3}$, is what B had at the end of the *second* year.

Then, by the problem, the Equation will be

$$\frac{5x-160}{3}=2\times\frac{2x+80}{3}.$$

From this Equation we shall find $x=\$320$.

25. What fraction is that, whose numerator being doubled, and denominator increased by 7, the value becomes $\frac{2}{3}$; but the denominator being doubled, and the numerator increased by 2, the value becomes $\frac{3}{5}$?

Let $\frac{x}{y}$ represent the fraction; then the Equations will be

$$\frac{2x}{y+7}=\frac{2}{3};\text{ and}$$

$$\frac{x+2}{2y}=\frac{3}{5}.$$

From these Equations we shall find $x=4$, and $y=5$. Hence the fraction is $\frac{4}{5}$.

26. A and B together can perform a piece of work in 8 days, A and C in 9 days, and B and C in 10 days. How many days would it take each person to perform the work alone?

Let x, y, and z respectively represent the number of days in which A, B, and C could do it; then

$\frac{1}{x}$ represents the part A could do in 1 day;

$\frac{1}{y}$ represents the part B could do in 1 day;

$\frac{1}{z}$ represents the part C could do in 1 day.

And, since A and B together, in 1 day, would do $\frac{1}{8}$ of the work, A and C together $\frac{1}{9}$ of it, and B and C together $\frac{1}{10}$ of it, the Equations will be

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{8};$$

$$\frac{1}{x}+\frac{1}{z}=\frac{1}{9};\text{ and}$$

$$\frac{1}{y}+\frac{1}{z}=\frac{1}{10}.$$

Subtracting the 2d Equation from the 1st, we shall have

$$\frac{1}{y}-\frac{1}{z}=\frac{1}{72}.$$

Adding this to the 3d Equation, we shall have

$$\frac{2}{y}=\frac{82}{720};$$

which will give $y=17\frac{23}{41}$. The values of x and z will now be found equal to $14\frac{34}{49}$, and $23\frac{7}{31}$, respectively.

27. From two places which are 154 miles apart, two persons set out at the same time to meet each other, one traveling at the rate of 3 miles in 2 hours, and the other at the rate of 5 miles in 4 hours; in how many hours will they meet?

Let x represent the number of hours; then, since

$\frac{3}{2}$ is the number of miles the *first* goes *per hour;* and

$\frac{5}{4}$ is the number of miles the *second* goes *per hour;*

$\frac{3x}{2}$ is the number of miles the *first* goes in x hours; and

$\frac{5x}{4}$ is the number of miles the *second* goes in x hours.

Then, since both together traveled 154 miles, the Equation will be

$$\frac{3x}{2}+\frac{5x}{4}=154;$$

which will give $x=56$ hours.

28. In a naval engagement, the number of ships captured was 7 more, and the number burned was 2 less, than the number sunk. Fifteen escaped, and the fleet consisted of 8 times the number sunk. Of how many ships did the fleet consist?

Let x represent the number of ships; then

$\frac{x}{8}$ represents the number *sunk;*

$\frac{x}{8}+7$ represents the number *captured;* and

$\frac{x}{8}-2$ represents the number *burned.*

We shall then have the Equation

$$\frac{x}{8}+\frac{x}{8}+7+\frac{x}{8}-2+15=x;$$

which will give $x=32$.

29. A and B together could have completed a piece of work in 15 days, but after laboring together 6 days, A was left to finish it alone, which he did in 30 days. In how many days could each have performed the work alone?

Let x represent the number of days in which A could do it, and y the number of days in which B could do it; then

$\frac{1}{x}$ is the part A could do in 1 day;

$\frac{1}{y}$ is the part B could do in 1 day; and

$\frac{1}{15}$ is the part *both* could do in one day.

The first Equation will then be

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{15}.$$

Now, since $\frac{6}{15}$ is the part done by A and B together in 6 days, $\frac{9}{15}$ is the part which A did in 30 days. Hence, 30 is $\frac{9}{15}$ of the number of days in which A *alone* could do the work.

The second Equation will then be

$$\frac{9x}{15}=30.$$

These two Equations will give $x=50$, and $y=21\frac{3}{7}$ days.

30. On comparing two sums of money, it is found that $\frac{2}{5}$ of the first is \$96 less than $\frac{3}{4}$ of the second, and that $\frac{5}{8}$ of the second is as much as $\frac{4}{9}$ of the first. What are the sums?

Let x represent the *first* sum, and y the *second*.

Then, by the conditions of the problem, the Equations will be

$$\frac{2x}{5}=\frac{3y}{4}-96;\ \text{and}$$

$$\frac{5y}{8}=\frac{4x}{9};$$

which will give $x=\$720$, and $y=\$512$.

31. A privateer, running at the rate of 10 miles an hour, discovers a ship 18 miles off, making way at the rate of 8 miles an hour. In how many hours will the ship be overtaken?

Let x represent the number of hours; then

$10x$ represents the number of miles the *privateer* runs from the commencement of the pursuit; and

$8x$ represents the number of miles the *ship runs* from the commencement of the pursuit.

But as the ship was 18 miles ahead at the commencement,

$8x+18$, also represents the number of miles the privateer runs before overtaking the ship.

The Equation will then be

$$10x=8x+18;$$

which will give $x=9$ hours.

32. In a composition of copper, tin, and lead, $\frac{1}{2}$ of the whole *minus* 16 pounds was copper, $\frac{1}{3}$ of the whole *minus* 12 pounds was tin, and $\frac{1}{4}$ of the whole *plus* 4 pounds was lead. What quantity of each was there in the composition?

Let x represent the *copper*, y the *tin*, and z the *lead*.

Then, by the conditions of the problem, the Equations will be

$$\frac{x+y+z}{2}-16=x;$$

$$\frac{x+y+z}{3}-12=y; \text{ and}$$

$$\frac{x+y+z}{4}+4=z.$$

These Equations will give $x=128$, $y=84$, and $z=76$ pounds.

33. The sum of \$660 was raised for a certain purpose by four persons, the first giving $\frac{1}{2}$ as much as the second, the third as much as the first and second, and the fourth as much as the second and third. What were the several shares contributed?

Let x, y, z, and w, respectively represent the several shares.

We shall then have the Equations

$$x+y+z+w=660;$$

$$x=\frac{y}{2};$$

$$z=x+y; \text{ and}$$

$$w=y+z.$$

These Equations will give $x=\$60$, $y=\$120$, $z=180$, and $w=\$300$.

34. Two pedestrians start from the same point, and travel in the same direction; the first steps twice as far as the second, but the second makes 3 steps while the first is making 2. How far has each gone when the first is 300 feet in advance of the second?

Let x represent the number of feet traveled by the *first*, and y the number traveled by the *second*.

Now, from the problem, it is evident that the first takes $\frac{2}{3}$ as many steps as the second; and, since the first steps twice as far as the second, the distance traveled by the first must be *twice* $\frac{2}{3}$, or $\frac{4}{3}$ of that traveled by the second; the first Equation is therefore

$$x = \frac{4y}{3}.$$

And since the first is 300 feet in advance of the second, the second Equation will be

$$x = y + 300.$$

These Equations will give $x = 1200$, and $y = 900$ feet.

35. A merchant has cloth at $3 a yard, and another kind at $5 a yard. How many yards of each kind must he sell to make 100 yards which shall bring him $450?

Let x represent the number of yards of the 1st kind, and y the number of the 2d kind; then

$3x$ represents the number of dollars received for the 1st kind; and

$5y$ represents the number of dollars received for the 2d kind.

The Equations will then be

$$x + y = 100; \text{ and}$$

$$3x + 5y = 450.$$

From these Equations we shall find $x = 25$, and $y = 75$ yards.

36. In the composition of a quantity of gunpowder, the nitre was 10 pounds more than $\frac{2}{3}$ of the whole, the sulphur $4\frac{1}{2}$ pounds less than $\frac{1}{6}$ of the whole, and the charcoal 2 pounds less than $\frac{1}{4}$ of the nitre. What was the amount of gunpowder?

Let x represent the number of pounds; then

$\frac{2x}{3} + 10$ represents the number of pounds of *nitre;*

$\frac{x}{6}-4\frac{1}{2}$ represents the number of pounds of *sulphur;* and

$\frac{1}{7}\left(\frac{2x}{3}+10\right)-2$, or $\frac{2x}{21}-\frac{4}{7}$, represents the number of pounds of *charcoal.*

The Equation will then be

$$\frac{2x}{3}+10+\frac{x}{6}-4\frac{1}{2}+\frac{2x}{21}-\frac{4}{7}=x;$$

which will give $x=69$ pounds.

37. Four places are situated in the order of the letters A, B, C, D. The distance from A to D is 34 miles; the distance from A to B is $\frac{2}{3}$ of the distance from C to D; and $\frac{1}{4}$ of the distance from A to B, *plus* $\frac{1}{2}$ of the distance from C to D, is 3 times the distance from B to C. What are the distances between A and B, B and C, C and D?

Let x represent the distance from A to B, y the distance from B to C, and z the distance from C to D.

Then, since the distance from A to D, or the whole distance from the *first* to the *last* place is 34 miles, the first Equation will be

$$x+y+z=34.$$

And by the problem, the other Equations will be

$$x=\frac{2z}{3}; \text{ and}$$

$$\frac{x}{4}+\frac{z}{2}=3y.$$

These three Equations will give $x=12$, $y=4$, and $z=18$ miles.

38. A vintner sold at one time 20 dozen of port wine, and 30 of sherry, for $120; and at another time 30 dozen of port, and 25 of sherry, at the same prices as before, for $140. What was the price of a dozen of each sort of wine?

Let x represent the price of the *port*, and y the price of the *sherry*, per dozen; then

$20x$ is what he received for the port at the 1st sale; and
$30y$ is what he received for the sherry at the 1st sale.

The first Equation will then be

$$20x+30y=120.$$

In a similar manner we shall have

$$30x+25y=140.$$

These Equations will give $x=\$3$, and $y=\$2$.

39. A person pays at one time, to two creditors, \$53, giving to one of them $\frac{4}{11}$ of the sum due to him, and to the other \$3 more than $\frac{1}{6}$ of his debt to him. At another time he pays them \$42, giving to the first $\frac{3}{7}$ of what remains due to him, and to the other $\frac{1}{3}$ of what remains due to him. What were the debts?

Let x represent the sum due to the *first*, and y the sum due to the *second;* then

$\frac{4x}{11}$ represents the 1st sum paid to the *first;* and

$\frac{y}{6}+3$ represents the 1st sum paid to the *second.*

The first Equation will then be

$$\frac{4x}{11}+\frac{y}{6}+3=53.$$

Now, $\frac{7x}{11}$ is the sum *remaining due* to the *first;* and

$\frac{5y}{6}-3$ is the sum *remaining due* to the *second;* then

$\frac{3}{7}\left(\frac{7x}{11}\right)$ represents the 2d sum paid to the *first;* and

$\frac{1}{3}\left(\frac{5y}{6}-3\right)$ represents the 2d sum paid to the *second.*

The second Equation will then be

$$\frac{3}{7}\left(\frac{7x}{11}\right)+\frac{1}{3}\left(\frac{5y}{6}-3\right)=42;$$

$$\text{or } \frac{3x}{11}+\frac{5y}{18}-1=42.$$

The two Equations will give $x=\$121$, and $y=\$36$.

40. A farmer has 86 bushels of wheat at 4*s.* 6*d.* per bushel, with which he wishes to mix rye at 3*s.* 6*d.* per bushel, and barley at 3*s.* per bushel, so as to make 136 bushels, that shall be worth 4*s.* a bushel. What quantity of rye and barley must he take?

Let x and y represent the number of bushels of rye and barley, respectively.

Now, 4*s*. 6*d*. is equal to $4\frac{1}{2}$ *shillings;*
3*s*. 6*d*. is equal to $3\frac{1}{2}$ *shillings;*
$86\times4\frac{1}{2}$, or 387, shillings is the value of the *wheat;*
$x\times3\frac{1}{2}$, or $\frac{7x}{2}$, is the value of the *rye, in shillings;*
$3y$ is the value of the barley in *shillings;* and
136×4, or 544 shillings, is the value of the *mixture.*

The two Equations will then evidently be

$$86+x+y=136; \text{ and}$$
$$387+\frac{7x}{2}+3y=544;$$

from which we shall find $x=14$, and $y=36$ bushels.

41. A composition of copper and tin, containing 100 cubic inches, weighs 505 ounces. How many ounces of each metal does it contain, supposing the weight of a cubic inch of copper to be $5\frac{1}{4}$ ounces, and of a cubic inch of tin $4\frac{1}{4}$ ounces?

Let x represent the number of ounces of *copper*, and y the number of ounces of *tin;* then we shall have

$$x+y=505.$$

Now, $x\div5\frac{1}{4}$, or $\frac{4x}{21}$, represents the *number* of *cubic inches* of copper; and

$y\div4\frac{1}{4}$, or $\frac{4y}{17}$, represents the *number* of *cubic inches* of tin.

The second Equation will then be

$$\frac{4x}{21}+\frac{4y}{17}=100.$$

From these Equations x will be found equal to 420 ounces, and y equal to 85 ounces.

42. A general, having lost a battle, found that he had only one half of his army *plus* 3600 men left fit for action; $\frac{1}{8}$ of his men *plus* 600 being wounded, and the rest, who were $\frac{1}{5}$ of his whole army, either slain, taken prisoners, or missing. Of how many men did the army consist?

Let x represent the number; then

$\frac{x}{8}+600$ represents the number *wounded;*

$\frac{x}{5}$ represents the number *slain*, taken *prisoners*, or *missing;* and

$\left(x-\frac{x}{8}-600-\frac{x}{5}\right)$, or $\frac{27x}{40}-600$, represents the number *remaining.*

Now, as only $\frac{1}{2}$ of his men plus 3600 were left, we shall have

$$\frac{27x}{40}-600=\frac{x}{2}+3600;$$

from which we shall find $x=24000$ men.

43. Two pipes, one of them running 5 hours, and the other 4, filled a cistern containing 330 gallons; and the same two pipes, the first running 2 hours, and the second 3, filled another cistern containing 195 gallons. How many gallons did each pipe discharge per hour?

Let x represent the number of gallons discharged by the *first,* and y the number discharged by the *second, per hour;* then

$5x$ is the number discharged by the 1st in 5 hours; and

$4y$ is the number discharged by the 2d in 4 hours.

The first Equation will then be

$$5x+4y=330;$$

and in a similar manner, we shall have

$$2x+3y=195;$$

from which Equations, we shall find $x=30$, and $y=45$ gallons.

44. After A and B had been employed on a piece of work for 14 days, they called in C, by whose aid it was completed in 28 days. Had C worked with them from the beginning, the work would have been completed in 21 days. In how many days would C alone have accomplished the work?

Let x represent the number of days; then

$\frac{1}{x}$ is the part C did in 1 day; and, since he was employed 14 days,

$\frac{14}{x}$ is the part of the work performed by C;

$1-\frac{14}{x}$, or $\frac{x-14}{x}$, is the part performed by A and B in 28 days;

$\frac{x-14}{28x}$ is the part A and B did in 1 day.

Hence, we have the Equation

$\frac{x-14}{28x}+\frac{1}{x}=\frac{1}{21}$, the part the three could do in 1 day.

This Equation will give $x=42$ days.

45. Some smugglers discovered a cave which would exactly hold their cargo, viz, 13 bales of cotton, and 33 casks of wine. A revenue cutter coming in sight while they were unloading, they sailed away with 9 casks, and 5 bales, leaving the cave two-thirds full. How many bales or casks would it contain?

Let x represent the number of *bales*, and y the number of *casks*, then $\frac{1}{x}$ represents the part of the cave which *one bale* would occupy, and $\frac{1}{y}$ represents the part which *one cask* would occupy.

The first Equation will therefore be

$$\frac{13}{x}+\frac{33}{y}=1, \text{ the } \textit{entire cave;}$$

and, as there were left in the cave 13—5, or 8 bales, and 33—9, or 24 casks, the second Equation will be

$$\frac{8}{x}+\frac{24}{y}=\frac{2}{3}.$$

Clearing these Equations of fractions, we have

$$13y+33x=xy;$$

$$24y+72x=2xy.$$

Multiplying the third Equation by 2,

$$26y+66x=2xy;$$

subtracting the fourth Equation from this, we have

$$2y-6x=0;$$

which will give $y=3x$.

Substituting $3x$ for y, in the third Equation, and dividing both sides of the resulting Equation by x, we shall have

$$30+33=3x;$$

which gives $x=24$.

The value of y is now easily found to be 72.

46. A gentleman left a sum of money to be divided among four servants, so that the share of the first was $\frac{1}{2}$ the sum of the shares of the other three; the share of the second was $\frac{1}{3}$ of the sum of the other three; and the share of the third $\frac{1}{4}$ of the sum of the other three; and it was also found that the share of the first exceeded that of the last by \$14. What was the whole sum? and the share of each?

Let x, y, z, and w, represent the several shares;
By the conditions of the problem, we shall then have

$$x=\frac{y+z+w}{2};$$

$$y=\frac{x+z+w}{3};$$

$$z=\frac{x+y+w}{4};\text{ and}$$

$$x=w+14.$$

These Equations will give $x=\$40$, $y=\$30$, $z=24$, and $w=\$26$.
The whole sum will then be $40+30+24+26=\$120$.

PROBLEMS

In Proportion, Percentage, Interest, &c.

1. Divide \$950 between two persons, so that their shares shall be to each other as 3 to 5.

Let x represent the share of the 1st, and y the share of the 2d; then we shall have

$$x+y=950;\text{ and}$$

$$x:y::3:5.$$

By converting this proportion into an Equation, we find

$$5x=3y,\ (149).$$

From this and the first Equation we shall find $x=\$356\frac{1}{4}$, and $y=\$593\frac{3}{4}$.

2. Find the Formulas for dividing any given sum s between two persons so that the shares shall be to each other as any two numbers a and b.

Let x and y represent the respective shares; then

$$x+y=s; \text{ and}$$
$$x:y::a:b.$$

This proportion converted into an Equation, gives

$$bx=ay, \text{ (149).}$$

From the two Equations we shall find $x=\frac{as}{a+b}$, and $y=\frac{bs}{a+b}$.

3. Divide the sum of \$3000 between A, B, and C, in the proportions of 1, 2, and 3.

Let x represent A's share; then

$$1:2::x:2x, \text{ B's share; and}$$
$$2:3::2x:3x, \text{ C's share (150).}$$

Then we shall have the Equation

$$x+2x+3x=3000;$$

from which we shall find $x=\$500$, A's share; hence $2\times500=\$1000$, B's share; and $3\times500=\$1500$, C's share.

4. Divide the sum of \$7600 between three persons, in the proportions of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{5}$.

Let x, y, and z, represent the several shares; then

$$x+y+z=7600;$$
$$x:y::\tfrac{1}{2}:\tfrac{1}{4}; \text{ and}$$
$$y:z::\tfrac{1}{4}:\tfrac{1}{5}.$$

The two proportions converted into Equations, will give

$$\frac{x}{4}=\frac{y}{2}; \text{ and}$$
$$\frac{y}{5}=\frac{z}{4}.$$

The three Equations will give $x=4000$, $y=2000$, and $z=1600$.

5. A bankrupt is indebted to A \$400, and to B \$700. He is able to pay both \$900. What sum should each of the two creditors receive?

Let x and y represent the respective shares; then we shall have

$$x+y=900; \text{ and}$$
$$x:y::400:700, \text{ or}$$
$$x:y::4:7, \text{ (158).}$$

This proportion will give

$$7x=4y.$$

From the two Equations x will be found equal to $\$327\frac{3}{11}$, and $y=\$572\frac{8}{11}$.

6. Three persons engaged in a speculation, towards which they contributed, respectively, \$300, \$400, and \$500. The profit amounted to \$550. What are the respective shares of profit?

Let x, y, and z, represent the respective shares of profit; then we shall have

$$x+y+z=550;$$
$$x:y::300:400; \text{ and}$$
$$y:z::400:500.$$

Simplifying these proportions (158), and multiplying together the *extremes* and *means*,

$$4x=3y; \text{ and}$$
$$5y=4z.$$

The values of x, y, and z, are now easily found equal to $\$137\frac{1}{2}$, $\$183\frac{1}{3}$, $\$229\frac{1}{6}$, respectively.

7. A, B, and C, in a joint mercantile adventure, lost \$742. A's part of the capital employed was to B's as 4 to 3, and B's was to C's as 5 to 6. What amount of loss should be borne by each?

Let x represent A's amount of loss; then

$$4:3::x:\frac{3x}{4}, \text{ B's amount of loss (150); and}$$

$$5:6::\frac{3x}{4}:\frac{18x}{20}, \text{ C's amount of loss, (150).}$$

We shall then have the Equation

$$x+\frac{3x}{4}+\frac{18x}{20}=742;$$

from which we shall find $x=\$280$, A's loss; hence $\frac{3}{4}$ of $280=210$, B's loss; and $\frac{18}{20}$ of $280=\$252$, C's loss.

8. Four persons rented a pasture, in which the first kept 8 oxen, the second 6, the third 10, and the fourth 12. The sum paid was \$40. What amount should have been paid by each person?

Let x, y, z, and w, represent the several shares of pay.

Then we shall have

$$x : y :: 8 : 6;$$
$$y : z :: 6 : 10;$$
$$z : w :: 10 : 12; \text{ and}$$
$$x+y+z+w=40.$$

The three proportions converted into Equations, give

$$6x=8y;$$
$$10y=6z; \text{ and}$$
$$12z=10w;$$

which, with the first Equation, will give $x=\$8\frac{8}{9}$, $y=\$6\frac{2}{3}$, $z=11\frac{1}{9}$, and $w=\$13\frac{1}{3}$.

9. A testator bequeathed his estate, amounting to $7830, to his three children, in such a manner that the share of the first was to that of the second as 2½ is to 2, and the share of the second to that of the third as 3½ to 3. What were the shares?

Let x, y, and z, represent the respective shares; then

$$x+y+z=7830;$$
$$x : y :: 2\tfrac{1}{2} : 2; \text{ and}$$
$$y : z :: 3\tfrac{1}{2} : 3.$$

These two proportions will give

$$2x=\frac{5y}{2}; \text{ and}$$
$$3y=\frac{7z}{2};$$

from which two Equations, with the first, we shall find $x=\$3150$, $y=\$2520$, and $z=2160$.

10. A, B, C, and D, together have $3000; A's part is to B's as 2 to 3. B and C together have $1500, and C's part is to D's as 3 to 4. What is the sum possessed by each person?

Let x, y, z, and w, represent the respective shares; then

$$x+y+z+w=3000;$$
$$x : y :: 2 : 3;$$
$$y+z=1500; \text{ and}$$
$$z : w :: 3 : 4.$$

Converting the two proportions into Equations, we shall have four Equations, from which we shall find $x=\$500$, $y=\$750$, $z=\$750$, and $w=\$1000$.

11. Three persons contributed funds in a joint speculation as follows: A $200 for 5 months, B $400 for 3 months, and C $500 for 4 months. The profit amounted to $600. What are the several shares of profit?

Let x, y, and z, represent the several shares of profit.

Now, each dollar contributed produced a profit *proportional to the time* it was in the business. Each person's share of profit is therefore proportional to his amount of *capital* × its *time;* in other words, the respective shares are to each other in the *compound ratio* of *capital* and *time*, (131).

Then we shall have

$$x : y :: 200 \times 5 : 400 \times 3;$$
$$x : z :: 200 \times 5 : 500 \times 4; \text{ and}$$
$$x + y + z = 600.$$

Simplifying the two proportions, we shall have

$$x : y :: 5 : 6; \text{ and}$$
$$x : z :: 1 : 2, \text{ (158)}.$$

From these proportions and the Equation, we shall find $x = \$142\frac{6}{7}$, $y = \$171\frac{3}{7}$, and $z = \$285\frac{5}{7}$.

12. Two persons rented a pasture for $43. The first put into it 100 sheep for 15 days, and the second 120 sheep for 9 days. What amount of rent should be paid by each person?

Let x and y represent the respective shares of rent to be paid by each; then

$$x : y :: 100 \times 15 : 120 \times 9; \text{ and}$$
$$x + y = 43.$$

The proportion simplified becomes

$$x : y :: 25 : 18;$$

from which and the Equation, we shall find $x = \$25$, and $y = \$18$.

13. A, B, and C, trade together; A ventures $1000 for 5 months, B $1200 for 4 months, and C $800 for 7 months. The profits of the partnership amount to $2310. What share of profit should be assigned to each?

Let x, y, and z, represent the several shares; then

$$x : y :: 1000 \times 5 : 1200 \times 4;$$
$$y : z :: 1200 \times 4 : 800 \times 7; \text{ and}$$
$$x + y + z = 2310.$$

Simplifying the two proportions, we have

$$x : y :: 25 : 24; \text{ and}$$
$$y : z :: 6 : 7;$$

which, with the Equation, will give $x = \$750$, $y = \$720$, and $z = \$840$.

14. An estate consisting of 1000 acres of land is to be divided between three persons, so that the first share shall be to the second as 2 to 3, and the first to the third as 1 to 2. What are the shares?

Let x, y, and z, represent the several shares; then

$$x : y :: 2 : 3;$$
$$x : z :: 1 : 2; \text{ and}$$
$$x + y + z = 1000.$$

From this Equation, and the two proportions, we shall find $x = 222\frac{2}{9}$, $y = 333\frac{1}{3}$, and $z = 444\frac{4}{9}$ acres.

15. Two men contracted to do a certain work for \$5000. In accomplishing the work, the first employed 100 laborers for 50 days; and the second 125 laborers for 60 days. To what shares of the stipulated sum are the two men respectively entitled?

Let x and y represent the respective shares; then

$$x : y :: 100 \times 50 : 125 \times 60; \text{ and}$$
$$x + y = 5000.$$

The proportion simplified becomes

$$x : y :: 2 : 3;$$

from which and the Equation we shall find $x = \$2000$, and $y = \$3000$.

16. A gentleman bequeathed \$18000 to his widow and his three sons, in the proportions of 2, $2\frac{1}{2}$, 3, and $3\frac{1}{2}$, respectively. His widow dying before the division was effected, the whole is to be divided proportionably among the three sons. What are their several shares?

Let x, y, and z, represent the several shares; then, as the widow died, the whole sum must be divided among the three sons in the proportions of $2\frac{1}{2}$, 3, and $3\frac{1}{2}$, respectively. Hence we shall have

$$x : y :: 2\tfrac{1}{2} : 3;$$
$$y : z :: 3 : 3\tfrac{1}{2}; \text{ and}$$
$$x + y + z = 18000.$$

From the two proportions, we shall find

$$3x=\frac{5y}{2}; \text{ and}$$
$$\frac{7y}{2}=3z.$$

From the three Equations, x will be found $=\$5000$, $y=\$6000$, and $z=\$7000$.

17. Find the Formulas for dividing between two partners the profits s of a joint adventure, in which the first had the capital a for the time b, and the second the capital c for the time d.

Let x, y, and z, represent the respective shares; then

$$x:y::ab:cd; \text{ and}$$
$$x+y=s.$$

From this proportion and Equation, we shall find $x=\frac{abs}{ab+cd}$, and $y=\frac{cds}{ab+cd}$.

Problems in Percentage.

18. A merchant finds that his capital, which is now \$12000, has increased in one year at the rate of 20 per cent. What was his capital at the beginning of the year?

Let x represent his capital at the beginning of the year; then

$$100:120::x:12000;$$

which will give $x=\$10000$

The solution of this problem will perhaps be more intelligible by forming an *Equation* directly from the conditions of the problem.

Let x represent his capital; then since

$\frac{20}{100}$ is the *ratio* of *percentage* (171),

$\frac{20x}{100}$ represents the *amount* of *percentage* on his *capital*, (172).

Then, since the capital *plus* the *percentage* on the capital is equal to \$12000, we shall have the Equation

$$x+\frac{20x}{100}=12000;$$

which will give $x=\$10000$.

19. What is the Formula for finding a sum of money which, increased at the rate of r *per cent*, shall amount to the sum a?

Let x represent the sum; then

$$100:100+r::x:a;$$

from which we shall find $x=\frac{100a}{100+r}$.

Or, let x represent the sum; then since

$\frac{r}{100}$ is the *ratio* of percentage (171),

$\frac{rx}{100}$ represents the *amount* of percentage on the sum, (172).

The Equation will then be

$$x+\frac{rx}{100}=a;$$

from which x will be found $=\frac{100a}{100+r}$.

20. An agent receives \$500, to be laid out in merchandise, after deducting his commission of $1\frac{1}{2}$ per cent on the amount of the purchase. What will be the amount of the purchase?

Let x represent the amount; then

$$100:101\tfrac{1}{2}::x:500;$$

from which x will be found $=\$492.61'$.

Or, let x represent the amount; then since

$1\frac{1}{2}\div 100$, or $\frac{3}{200}$, is the *ratio* of percentage,

$\frac{3x}{200}$ represents the *amount* of percentage on the purchase.

Then we shall have the Equation

$$x+\frac{3x}{200}=500.$$

This Equation will give $x=\$492.61'$.

21. A merchant obtains an insurance at 2 per cent on a stock of goods valued at $7500, which includes this amount and the premium for the insurance. What is the sum insured?

Let x represent the sum; then

$$100 : 100-2 :: x : 7500.$$

This proportion will give $x=7653.06'$.

Or, let x represent the sum; then since

$\frac{2}{100}$ is the *ratio* of percentage,

$\frac{2x}{100}$ represents the *premium* for the insurance.

We shall then have the Equation

$$x-\frac{2x}{100}=7500.$$

From this Equation we shall find $x=\$7653.06'$.

22. What is the Formula for finding a sum of money which, diminished at the rate of r per cent, shall leave the sum a?

Let x represent the sum; then

$$100 : 100-r :: x : a.$$

This proportion will give $x=\frac{100a}{100-r}$.

Or, let x represent the sum; then since

$\frac{r}{100}$ is the *ratio* of percentage,

$\frac{rx}{100}$ represents the *amount* of percentage on the sum

The Equation will then be

$$x-\frac{rx}{100}=a;$$

which will give $x=\frac{100a}{100-r}$.

23. The profits of a manufacturing company this year amount to $3096, which is $3\frac{1}{4}$ per cent less than their profits last year. What was the amount of profits last year?

Let x represent the profits last year; then

$$100 : 100-3\tfrac{1}{4} :: x : 3096.$$

This proportion will give $x=\$3200$.

Or, let x represent the profits last year; then since

$3\tfrac{1}{4} \div 100$, or $\frac{13}{400}$, is the *ratio* of percentage,

$\frac{13x}{400}$ represents the *percentage* on their profits last year.

Then the Equation will be

$$x-\frac{13x}{400}=3096;$$

from which we shall find $x=\$3200$.

24. What must be the percentum of profit at which a quantity of merchandise, bought for $3750, must be sold, that the whole amount of profit shall be $1500?

Let x represent the percentum of profit; then

$$100 : x :: 3750 : 1500;$$

which will give $x=40$ per cent.

Or, let x represent the percentum of profit; then since

$\frac{x}{100}$ is the *ratio* of percentage,

$\frac{3750x}{100}$ is the *amount* of percentage on $3750.

Then we shall have the Equation

$$\frac{3750x}{100}=1500;$$

from which we shall find $x=40$ per cent.

25. What is the Formula for finding the *rate per cent* at which the sum s must be increased to produce the sum a?

Let x represent the *rate per cent;* then

$$100 : 100+x :: s :: s\,a;$$

from which we shall find $x=\frac{100(a-s)}{s}$.

Or, let x represent the *rate per cent;* then since

$\frac{x}{100}$ is the *ratio* of percentage,

$\frac{sx}{100}$ represents the *amount* of percentage on the sum s.

The Equation will then be

$$s+\frac{sx}{100}=a.$$

26. A quantity of silk was purchased for \$220, and, on account of its having become damaged, was sold for \$176. What was the percentum of loss sustained?

Let x represent the percentum of loss; then as $220-176=44$ dollars is the loss sustained, we shall have

$$100:x::220:44.$$

This proportion will give $x=20$ per cent.

Or, let x represent the percentum of loss; then since

$\frac{x}{100}$ is the ratio of percentage,

$\frac{220x}{100}$ represents the percentage on the cost of the silk.

The Equation will then be

$$\frac{220x}{100}=44;$$

which will give $x=20$ per cent.

27. What is the Formula for finding the *rate per cent* at which the sum s must be diminished to leave the sum a?

Let x represent the *rate per cent;* then

$$100:100-x::s:a.$$

From this proportion we shall find $x=\frac{100(s-a)}{s}$.

Or, let x represent the *rate per cent;* then since

$\frac{x}{100}$ is the *ratio* of percentage,

$\frac{sx}{100}$ represents the percentage on the sum s.

We shall then have

$$s-\frac{sx}{100}=a;$$

from which we shall find $x=\frac{100(s-a)}{s}$.

28. What amount of stock in an Insurance Office, at a discount of 5 per cent, could be purchased for $3800 ?

Let x represent the amount; then

$$100:100-5::x:3800;$$

which will give $x=\$4000$.

Or, let x represent the amount; then since

$\frac{5}{100}$ is the ratio of percentage,

$\frac{5x}{100}$ represents the percentage on the *amount*.

The Equation will then be

$$x-\frac{5x}{100}=3800;$$

which will give $x=\$4000$.

29. A merchant finds that his capital, which is now $4350, has decreased in one year at the rate of 12½ per cent. What was his capital at the beginning of the year?

Let x represent his capital at the beginning of the year; then

$$100:100-12\tfrac{1}{2}::x:4350;$$

from which x will be found $=\$4971.42'$.

Or, let x represent his capital at the beginning of the year; then since

$12\frac{1}{2}\div 100$, or $\frac{25}{200}$, is the ratio of percentage,

$\frac{25x}{200}$ represents the percentage on his original capital.

The Equation will then be

$$x - \frac{25x}{200} = 4350;$$

which will give $x = \$4971.42'$.

30. What amount of stock in a manufacturing establishment, at an advance of $6\frac{1}{2}$ per cent, could be purchased for $1200?

Let x represent the amount; then

$$100 : 100 + 6\tfrac{1}{2} :: x : 1200.$$

This proportion will give $x = \$1126.76'$.

Or, let x represent the amount; then since

$6\frac{1}{2} \div 100$, or $\frac{13}{200}$ is the *ratio* of percentage,

$\frac{13x}{200}$ represents the *percentage* on the *amount.*

We shall then have the Equation

$$x + \frac{13x}{200} = 1200;$$

which will give $x = \$1126.76'$.

31. A quantity of damaged cloth was sold for $250, which was at a loss of $16\frac{2}{3}$ per cent. For what sum was the cloth purchased?

Let x represent the sum; then

$$100 : 100 - 16\tfrac{2}{3} :: x : 250.$$

This proportion will give $x = \$300$.

Or, let x represent the sum; then since

$16\frac{2}{3} \div 100$, or $\frac{50}{300}$, is the *ratio* of percentage,

$\frac{50x}{300}$ represents the *percentage* on the cost.

Hence, we shall have the following Equation:

$$x - \frac{50x}{300} = 250;$$

from which we shall find $x = \$300$.

32. The population of a city increased from 7850 to 11775 inhabitants, in one year. What was the percentum of increase during the year?

Let x represent the *percentum* of *increase*; then since the population increased $11775-7850=3925$ inhabitants in one year, we shall have the proportion

$$100 : x :: 7850 : 3925;$$

from which x will be found equal to 50 per cent.

Or, let x represent the *percentum* of *increase;* then since

$\frac{x}{100}$ is the *ratio* of percentage,

$\frac{7850x}{100}$ represents the *amount* of *percentage* on 7850.

Then, we shall have the Equation

$$\frac{7850x}{100}=3925;$$

which will give $x=50$ per cent.

33. An agent receives \$2030 to invest in merchandise—himself to retain a commission of $1\frac{1}{2}$ per cent on the amount of the purchase What is the sum to be invested?

Let x represent the sum; then

$$100 : 100+1\tfrac{1}{2} : x :: 2030;$$

from which we shall find $x=\$2000$.

Or, let x represent the sum; then since

$1\frac{1}{2}\div 100$, or $\frac{3}{200}$ is the *ratio* of percentage,

$\frac{3x}{200}$ represents the merchant's *commission.*

We shall then have the Equation

$$x+\frac{3x}{200}=\overline{2030};$$

which will give $x=\$2000$.

34. A merchant wishes to effect an insurance on a stock of goods amounting to \$3573, which shall cover both the value of the goods and the premium of insurance. What is the sum to be insured, allowing the rate to be $\frac{3}{4}$ per cent?

Let x represent the sum; then

$$100 : 100-\tfrac{3}{4} :: x : 3573;$$

which will give $x=\$3600$.

Or, let x represent the sum; then since

$\frac{3}{4} \div 100$, or $\frac{3}{400}$, is the *ratio* of percentage,

$\frac{3x}{400}$ represents the *premium* of *insurance.*

Then, we shall have the following Equation:

$$x-\frac{3x}{400}=3573;$$

from which we shall find $x=\$3600$.

35. What amount of stock in a Savings Bank, at an advance of 5 per cent, could be purchased for \$4200? and what amount in another, at a discount of 5 per cent, could be purchased for \$1995?

1st. Let x represent the amount; then

$$100 : 105 :: x : 4200;$$

from which x will be found $=\$4000$.

Or, let x represent the amount; then since

$\frac{5}{100}$ is the *ratio* of percentage,

$\frac{5x}{100}$ represents the *percentage* on the stock.

We shall then have the Equation

$$x+\frac{5x}{100}=4200.$$

This Equation will give $x=\$4000$.

2d. Let x represent the amount; then

$$100 : 100-5 :: x : 1995;$$

from which we shall find $x=\$2100$.

Or, let x represent the amount; then since

$\frac{5}{100}$ is the *ratio* of percentage,

$\frac{5x}{100}$ represents the *percentage* on the stock.

We shall then have the Equation

$$x-\frac{5x}{100}=1995;$$

which will give $x=\$2100$.

Problems in Interest, &c.

36. What principal would amount to \$1000 in 5 years allowing the rate of interest to be 6 per cent?

Let x represent the principal; then

$\frac{6x}{100}$ represents the *interest* for *one* year (174); and

$\frac{30x}{100}$ represents the interest for *five* years.

The Equation will then be

$$x+\frac{30x}{100}=1000;$$

which will give $x=\$769\frac{3}{13}$.

37. What is the Formula for finding the principal which, at interest at r per cent, would amount to the sum a in t years?

Let x represent the *principal;* then

$\frac{rx}{100}$ represents the *interest* for *one* year; and

$\frac{trx}{100}$ represents the interest for t years.

We shall then have the Equation

$$x+\frac{trx}{100}=a;$$

from which we shall find $x=\frac{100a}{100+rt}$.

38. What principal would amount to \$2500, in 10 years; allowing the rate of interest to be 7 per cent?

Let x represent the principal; then

$\frac{7x}{100}$ represents the *interest* for one year; and

$\frac{70x}{100}$ represents the interest for ten years.

The Equation will then be

$$x+\frac{70x}{100}=2500;$$

from which we shall find $x=\$1470.588'$.

39. At what rate per cent must \$1000 be put on interest to amount to \$1150 in 2 years and 6 months?

Let x represent the *rate per cent;* then

$\frac{1000x}{100}$ represents the *interest* for *one* year; and

$\frac{1000x}{100}\times 2\frac{1}{2}$, or $\frac{5000x}{200}$, represents the interest for $2\frac{1}{2}$ years,

We shall then have the Equation

$$1000+\frac{5000x}{200}=1150;$$

from which we shall find $x=6$ per cent.

40. What is the Formula for finding the rate per cent of interest at which the sum s would amount to the sum a in t years?

Let x represent the *rate per cent;* then

$\frac{sx}{100}$ represents the *interest* for one year; and

$\frac{stx}{100}$ represents the interest for t years.

The Equation will then be

$$s+\frac{stx}{100}=a;$$

from which x will be found $=\frac{100(a-s)}{st}$.

41 In how many years would \$6000 amount to \$7470, allowing the rate of interest to be 7 per cent?

Let x represent *the number of years;* then

$6000\times\frac{7}{100}$, or $\frac{42000}{100}$, is the *interest* for *one* year; and

$\frac{42000x}{100}$ represents the interest for x years.

We shall then have the following Equation:

$$6000+\frac{42000x}{100}=7470;$$

from which we shall find $x=3\frac{1}{2}$ years.

42. What is the Formula for finding the time in which the sum s would amount to the sum a, if the interest be at r per cent?

Let x represent the *number of years;* then

$\frac{sr}{100}$ represents the *interest* for one year; and

$\frac{srx}{100}$ represents the interest for x years.

The Equation will then be

$$s+\frac{srx}{100}=a;$$

which will give $x=\frac{100(a-s)}{sr}$.

43. What principal would produce as much interest in $3\frac{1}{2}$ years, as \$500 would in 4 years, the rate of interest in both cases being 6 per cent?

Let x represent the *principal;* then

$\frac{6x}{100}$ represents the *interest* for *one* year; and

$\frac{42x}{200}$ represents the interest for $3\frac{1}{2}$ years; and

120 dollars is the interest on \$500 for 4 years.

Then the Equation will be

$$\frac{42x}{200}=120;$$

from which x will be found $=\$571\frac{3}{7}$.

44. At what rate per cent of interest would \$525 produce the same amount of interest in 5 years, that \$700 would produce at 5 per cent in 3 years?

Let x represent the *rate per cent;* then

$\frac{525x}{100}$ represents the *interest* on \$525 for one year; and

$\frac{2625x}{100}$ is the interest on \$525 for 5 years; and

105 dollars is the interest on \$700 for 3 years at 5 per cent.

We shall then have the Equation

$$\frac{2625x}{100}=105.$$

This Equation will give $x=4$ per cent.

45. A person who possessed a capital of \$70000, put the greater part of it at interest at 5 per cent, and the other part at 4 per cent. The interest on the whole was \$3250 per annum. Required the two parts.

Let x represent the *greater* part, and y the less; then we shall have for the first Equation

$$x+y=70000.$$

Now, $\frac{5x}{100}$ represents the *interest* on the *greater* part for one year, at 5 per cent; and

$\frac{4y}{100}$ represents the interest on the *less* part for one year, at 4 per cent.

The second Equation will then be

$$\frac{5x}{100}+\frac{4y}{100}=3250.$$

From the two Equations we shall find $x=\$45000$, and $y=\$25000$.

46. The sum of \$200 is to be applied in part towards the payment of a debt of \$300, and in part to paying the interest at 6 per cent *in advance*, for 12 months, on the remainder of the debt. What is the amount of the payment that can be made on the debt?

Let x represent the *payment;* then

$300-x$ represents the *remainder* of the debt; and

$(300-x)\,\frac{6}{100}$ represents the *interest* on the remainder.

We shall then have the Equation

$$x+\ (300-x)\,\frac{6}{100}\ =200\,;$$

from which we shall find $x=\$193.61'$.

47. A is indebted to B $1000, and is able to raise but $600. With this sum A proposes to pay part of the debt, and the interest, at 8 per cent in advance, on his note at 2 years for the remainder. For what sum should the note be drawn?

Let x represent the sum; then

$\frac{8x}{100}$ represents the *interest* on the note for *one* year;

$\frac{16x}{100}$ represents the interest on the note for 2 years; and

$1000-x$ represents the *present* payment.

Therefore, the Equation is

$$(1000-x)+\ \frac{16x}{100}\ =600.$$

From this Equation x will be found equal to $476.19'.

48. Find the Formulas for dividing the sum s into two parts, one of which is to be applied towards the payment of a debt of n dollars, and the other to paying the interest, in advance, on the remainder of the debt, for t years, at r per cent per annum?

Let x represent the *present payment;* then

$n-x$ represents the *remainder* of the debt; and

$(n-x)\,\frac{rt}{100}$ represents the *interest* on the remainder for t years, at r per cent.

Hence, the Equation will be

$$x+(n-x)\ \frac{rt}{100}\ =s.$$

From this Equation, x will be found $=\frac{100s-nrt}{100-rt}$, the *present* payment. Hence, $s-\frac{100s-nrt}{100-rt}=\frac{rt(n-s)}{100-rt}$, the *interest* on the *remainder*.

Problems in Progressions.

1. The first term of an increasing Arithmetical Progression is 3, the common difference of the terms is 2, and the number of terms 20. What is the last term? and the sum of all the terms?

The last term will be

$$3+2(20-1)=41, (176).$$

The sum of all the terms will be

$$\tfrac{1}{2}(3+41)\times 20=440, (180).$$

2. The first term of a decreasing Arithmetical progression is 100, the common difference of the terms is 3, and the number of terms 34. What is the last term? and the sum of all the terms?

The last term will be

$$100-3(34-1)=1, (176).$$

The sum of all the terms will be

$$\tfrac{1}{2}(100+1)\times 34=1717, (180).$$

3. What is the sum of the numbers 1, 2, 3, 4, 5, &c., continued to 1000 terms?

Here, the common difference is evidently 1; the number of terms 1000; and the last term 1000.
Hence, the sum of all the terms will be

$$\tfrac{1}{2}(1+1000)\times 1000=500500, (180).$$

4. What is the *common difference* of the terms in an Arithmetical progression whose first term is 10, last term 150, and number of terms 21?

The common difference is

$$(150-10)\div(21-1)=7, (177).$$

5. If the *third* term of an Arithmetical progression be 40, and the fifth term 70, what will the *fourth* term be?

The fourth term will be

$$\tfrac{1}{2}(40+70)=55, \ (179).$$

6. If the first term of an Arithmetical progression be 5, and the fifth term 30, what will the *second*, *third*, and *fourth* terms be?

The *common difference* of the terms is

$$(30-5)\div(5-1)=6\tfrac{1}{4}, \ (177).$$

Hence, the second term will be $5+6\frac{1}{4}=11\frac{1}{4}$; the third $11\frac{1}{4}+6\frac{1}{4}=17\frac{1}{2}$; and the fourth $17\frac{1}{2}+6\frac{1}{4}=23\frac{3}{4}$.

7. If the fourth term of an Arithmetical progression be 37, and the eighth term 60, what are the intermediate terms?

The *number of terms* being 5, the *common difference* is

$$(60-37)\div(5-1)=5\tfrac{3}{4}, \ (177)$$

Hence, the fifth term is $37+5\frac{3}{4}=42\frac{3}{4}$; the sixth is $42\frac{3}{4}+5\frac{3}{4}=48\frac{1}{2}$; the seventh $48\frac{1}{2}+5\frac{3}{4}=54\frac{1}{4}$.

8. What is the sum of 25 terms of an increasing Arithmetical progression in which the first term is $\frac{1}{2}$, and the common difference of the terms also $\frac{1}{2}$?

The *last term* will be

$$\tfrac{1}{2}+\tfrac{1}{2}(25-1)=12\tfrac{1}{2}, \ (176).$$

Hence, the sum of all the terms is

$$\tfrac{1}{2}(\tfrac{1}{2}+12\tfrac{1}{2})\times 25=162\tfrac{1}{2}, \ (180).$$

9. The first term of an increasing Arithmetical progression is 1, and the number of terms is 23. What must the *common difference* be, that the sum of all the terms may be 100?

Let x represent the common difference; then

$1+22x$ represents the last term (176); and

$\dfrac{2+22x}{2}\times 23$ represents the sum of the terms, (180).

We shall then have the Equation

$$\frac{2+22x}{2}\times 23=100\,;$$

which will give $x=\frac{7}{23}$

10. If the first term of a decreasing Arithmetical progression is 100, and the number of terms 21, what must the common difference be that the sum of the series may be 1260?

Let x represent the common difference; then

$100-20x$ represents the last term (176); and

$\frac{200-20x}{2}\times 21$ represents the sum of the terms, (180).

We shall then have the following Equation:

$$\frac{200-20x}{2}\times 21=1260\,;$$

from which we shall find $x=4$.

11. A and B start together and travel in the same direction. A goes 40 miles per day; B goes 20 miles the first day, and increases his rate of travel $\frac{3}{4}$ of a mile per day. How far will they be apart at the end of 40 days?

$40\times 40=1600$, is the number of miles A will travel in 40 days;

$20+\frac{3}{4}(40-1)=49\frac{1}{4}$, is the number of miles B will travel on the *last day* (176);

$\frac{1}{2}(20+49\frac{1}{4})\times 40=1385$, is the whole distance B will travel, (180).

Hence, $1600-1385=215$ miles is the distance they are apart at the end of 40 days.

12. One hundred stones being placed on the ground in a straight line, at a distance of 2 yards from each other; how far will a person travel who shall bring them, one by one, to a basket which is placed 2 yards from the first stone?

It is evident that the person will travel 4 yards in bringing the first stone to the basket, 8 yards in bringing the second, 12 yards in bringing the third, &c.

Hence, the first term of the series is 4, the common difference 4, and the number of terms 100.

Then, the last term of the series is

$4+4(100-1)=400$ yards, the distance he travels in bringing the *last* stone to the basket, (176).

Hence, the sum of all the terms will be

$$\tfrac{1}{2}(4+400)\times 100=20200 \text{ yards}=11 \text{ miles } 840 \text{ yards},$$

the *whole* distance he will travel, (180).

13. Find the third term of an harmonical progression whose first and second terms are 12 and 15 respectively.

Let x represent the third term; then

$$12 : x :: 15-12 : x-15, \ (182).$$

This proportion will give $x=20$, (170).

14. What is the first term of an harmonical progression whose second and third terms are 30 and 20 respectively?

Let x represent the first term; then

$$x : 20 :: x-30 : 30-20 \ (182);$$

from which we shall find $x=60$, (170).

15. What is the *fourth* term of an harmonical *proportion* whose first, second, and third terms are 2, 3, and 8 respectively?

Let x represent the fourth term; then

$$2 : x :: 3-2 : x-8 \ (182);$$

from which we shall find $x=16$.

16. If the first and third terms of an harmonical progression be 25 and 40 respectively, what will the second term be?

Let x represent the second term; then

$$25 : 40 :: x-25 : 40-x, \ (182).$$

From this proportion, we shall find $x=30\tfrac{10}{13}$.

17. The first and fourth terms of an harmonical progression are 10 and 20 respectively. What are the two intermediate terms?

We first find two *arithmetical means* between $\tfrac{1}{10}$ and $\tfrac{1}{20}$. The common difference is

$$\left(\frac{1}{10}-\frac{1}{20}\right) \div (4-1)=\frac{1}{60}, \ (177).$$

Hence, $\dfrac{1}{10}-\dfrac{1}{60}=\dfrac{1}{12}$, and $\dfrac{1}{12}-\dfrac{1}{60}=\dfrac{1}{15}$, are the two *arithmetical means*.

The *reciprocals* of these two means are 12 and 15, (80); the two intermediate terms are, then, 12 and 15, respectively, (183).

18. The fifth and eighth terms of an harmonical progression are 20 and 40 respectively. What are the two intermediate terms?

We find two arithmetical means between $\frac{1}{20}$ and $\frac{1}{40}$. The common difference is

$$\left(\frac{1}{20}-\frac{1}{40}\right)\div(4-1)=\frac{1}{120}, \text{ (177).}$$

Therefore, $\frac{1}{20}-\frac{1}{120}=\frac{1}{24}$, and $\frac{1}{24}-\frac{1}{120}=\frac{1}{30}$, are the respective *arithmetical means* between $\frac{1}{20}$ and $\frac{1}{40}$.

The reciprocals of these terms are 24 and 30, respectively (80).
Therefore, 24 and 30, are the two intermediate terms, (183).

19. The first term of a Geometrical progression is 2, the ratio of the progression is 3, and the number of terms 4. What is the last term? and the sum of all the terms?

The last term is $2\times 3^{4-1}=54$, (186).
The sum of all the terms is

$$(54\times 3-2)\div(3-1)=80, \text{ (190).}$$

20. The first term of a Geometrical progression is $\frac{1}{2}$, the ratio of the progression is $\frac{1}{3}$, and the number of terms 4. What is the last term? and the sum of all the terms?

By raising the ratio to the *third* power, we have

$$\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}=\frac{1}{27}.$$

Then the last term will be

$$\frac{1}{2}\times\frac{1}{27}=\frac{1}{54}, \text{ (186).}$$

The sum of all the terms will be

$$\left(\frac{1}{2}-\frac{1}{54}\times\frac{1}{3}\right)\div\left(1-\frac{1}{3}\right)=\frac{20}{27}, \text{ (190).}$$

21. What is the sum of an infinite number of terms in the Geometrical progression whose first term is 100, and ratio $\frac{1}{4}$?

The sum of the terms is

$$100 \div \left(1-\frac{1}{4}\right)=133\frac{1}{3}, \text{ (191)}.$$

22. What is the sum of an infinite number of terms in the Geometrical progression whose first term is 300, and ratio $\frac{1}{3}$?

The sum of an infinite number of terms will be

$$300 \div \left(1-\frac{1}{3}\right)=450, \text{ (191)}.$$

23. If the first and third terms of a Geometrical progression are 8 and 72, respectively, what is the second term?

The second term is $\sqrt{72\times 8}=24$, (189).

24. If the third and fifth terms of a Geometrical progression be 75 and 300, respectively, what will the fourth term be?

The fourth term will be

$$\sqrt{300\times 75}=150, \text{ (189)}.$$

25. If the first and fourth terms of a Geometrical progression are 3 and 24, respectively, what are the two intermediate terms?

$24\div 3=8$ is the *cube of the ratio*, (187).

Hence, the ratio is 2. Then $3\times 2=6$ is the second term, and $6\times 2=12$ is the third term.

26. If the seventh and tenth terms of a Geometrical progression are 6 and 750, respectively, what are the intermediate terms?

Taking the 7th term for the *first;* the number of terms will be

$750\div 6=125$, is the *cube of the ratio*, (187).

Then, 5 is the ratio; and $6\times 5=30$ is the eighth term; and $30\times 5=150$ is the ninth term.

27. What is the sum of an infinite number of terms in the series 1, $\frac{1}{2}$, $\frac{1}{4}$, &c., in which the ratio of the progression is evidently $\frac{1}{2}$?

The sum will be $1 \div \left(1 - \frac{1}{2}\right) = 2$, (191).

28. If a body move forever at the rate of 2000 feet the first second, 1000 the second, 500 the third, and so on, what is the utmost distance it can reach?

In this problem the ratio is $\frac{1}{2}$; and the sum of an infinite number of terms in the series is

$$2000 \div \left(1 - \frac{1}{2}\right) = 4000 \text{ feet.}$$

29. If 10 yards of cloth be sold at the rate of $1 for the first yard, $2 for the second, $4 for the third, and so on, what would be the price of the last yard? and what would the whole amount to?

Here, the ratio is 2; then the price of the last yard will be

$$1 \times 2^{-9} = 512 \text{ dollars, (186).}$$

The cost of the whole will be

$$(512 \times 2 - 1) \div (2 - 1) = 1023 \text{ dollars, (190).}$$

30. If 13 acres of land were purchased at the rate of $2 for the first acre, $6 for the second, $18 for the third, and so on, what would the last acre amount to?

The cost of the last acre would be

$$2 \times 3^{12} = \$1062882, \text{ (186).}$$

31. Allowing the interest of a sum of money to be $500 the first year, $400 the second, $320 the third, and so on, forever, what would be the whole amount of interest?

Here, the ratio is $\frac{400}{500}$, or $\frac{4}{5}$. Hence, the whole amount of interest will be

$$500 \div \left(1 - \frac{4}{5}\right) = 2500 \text{ dollars, (191).}$$

32. Two bodies move at the same time from the same point, in opposite directions. One goes 2 miles the first hour, 4 the second, 6 the third, and so on; the other goes 2 miles the first hour, 4 the second, 8 the third, &c. How far will they be apart at the end of 12 hours?

The first body increases its rate of traveling 2 miles every hour; and the second *doubles* its rate every hour; then

$2+2(12-1)=24$, is the number of miles the first went in the last hour (176); and

$\frac{1}{2}(2+24)\times 12=156$, is the whole number of miles the first went, (180). Also,

$2\times 2^{11}=4096$, is the number of miles the second went in the last hour (186); and

$(4096\times 2-2)\div(2-1)=8196$, is the whole number of miles the second went.

Therefore, $156+8190=8346$ miles, the distance between them.

33. A and B set out at the same time to meet each other. A travels 3, 4, 5, &c., miles on successive days, and B 3, $4\frac{1}{2}$, $6\frac{3}{4}$, &c., miles on successive days. They meet in 10 days. What is the distance between the two places from which they traveled?

In the *first* series of numbers the *common difference* is 1; and in the *second* series, the *ratio* is $\frac{4\frac{1}{2}}{3}=1\frac{1}{2}$.

A traveled on the last day

$$3+1(10-1)=12 \text{ miles, (176).}$$

A traveled altogether $\frac{(3+12)}{2}\times 10=75$ miles, (180).

B traveled on the last day

$$3\times(1\tfrac{1}{2})^9=\frac{59049}{512} \text{ miles, (186).}$$

B traveled altogether

$$\left(\frac{59049}{512}\times 1\tfrac{1}{2}-3\right)\div(1\tfrac{1}{2}-1)=339\tfrac{507}{512} \text{ miles, (190).}$$

Therefore, $75+339\frac{507}{512}=414\frac{507}{512}$ miles is the distance between the places.

PROBLEMS

In Permutations and Combinations.

1. In how many different ways might a company of 10 persons seat themselves around a table?

Substituting 10 for n in the Formula (194), we have

$$10\times9\times8\times7\times6\times5\times4\times3\times2\times1=3628800 \text{ ways.}$$

2. How many different numbers might be expressed by the 10 numeral figures, if 5 figures be used in each number?

$$10\times9\times8\times7\times6=30240, \ (194).$$

3. In how many different ways may the names of the 12 months of the year be arranged one after another?

$$12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1=479001600, \ (194).$$

4. How many different permutations of 8 men could be formed out of a company consisting of 15 men?

$$15\times14\times13\times12\times11\times10\times9\times8=259459200, \ (194).$$

5. In how many different ways might the seven prismatic colors, *red*, *orange*, *yellow*, *green*, *blue*, *indigo*, and *violet*, have been arranged in the solar spectrum?

$$7\times6\times5\times4\times3\times2\times1=5040, \ (194).$$

6. How many different *combinations* of *two colors* could be formed out of the seven prismatic colors?

$$\frac{7\times6}{1\times2}=21, \ (196).$$

7. How many different combinations of 5 letters may be formed out of the 26 letters of the Alphabet?

$$\frac{26\times25\times24\times23\times22}{1\times2\times3\times4\times5}=65780, \ (196).$$

8. How many different combinations of 2 *elements* might be formed out of the 56 *elements* described in Chemistry?

$$\frac{56\times55}{1\times2}=1540, \ (196).$$

9. In how many different ways might a company of 20 men be arranged, in single file, in a procession?

$$20\times19\times18\times17\times16\times15\times14\times13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1=2432902008176640000, (194).$$

10. A farmer wishes to select a team of 6 horses out of a drove containing 10 horses. How many different choices for the team will he be able to make?

$$\frac{10\times9\times8\times7\times6\times5}{1\times2\times3\times4\times5\times6}=210, (196).$$

11. In how many different ways might the planets Mercury, Venus, the Earth, Mars, Jupiter, Saturn, Uranus, and Neptune, succeed one another in the solar system?

$$8\times7\times6\times5\times4\times3\times2\times1=40320, (194).$$

12. A company of 20 persons engaged to remain together so long as they might be able to combine in different couples in their evening walks. What time will be required to fulfil the engagement?

$$\frac{20\times19}{1\times2}=190 \text{ days}, (196).$$

13. How many different permutations of 7 letters might be formed out of the 26 letters of the Alphabet?

$$26\times25\times24\times23\times22\times21\times20=3315312000, (194).$$

14. In an exhibition of a Public School, 5 speakers are to be taken from a class of 15 students. How many different selections of the 5 might be made? and in how many different ways might the 5 succeed one another in the delivery of their speeches?

$$\frac{15\times14\times13\times12\times11}{1\times2\times3\times4\times5}=3003 \text{ selections}, (196).$$

$$5\times4\times3\times2\times1=120 \text{ ways}, (194).$$

15. Out of a company consisting of 100 soldiers, six are to be taken for a particular service. How many different selections of the 6 might be made? and in how many different ways might the 6 chosen be disposed with regard to the order of succession?

$$\frac{100\times99\times98\times97\times96\times95}{1\times2\times3\times4\times5\times6}=1192052400 \text{ selections}, (196).$$

$$6\times5\times4\times3\times2\times1=720 \text{ ways}, (194).$$

EXERCISES

On Rationalization and Pure Equations.

1. Find the value of x in the Equation

$$24-\sqrt{2x^2+9}=15.$$

By transposition, $-\sqrt{2x^2+9}=15-24$.

Adding similar terms and squaring both sides,

$$2x^2+9=81\,;$$

from which we shall find $x=\pm 6$, (256).

2. Find the value of x in the Equation

$$13-\sqrt{3x^2+16}=5.$$

By transposition, $-\sqrt{3x^2+16}=5-13$.

Adding similar terms and squaring both sides,

$$3x^2+16=64\,;$$

from which we shall find $x=\pm 4$, (256).

3. Find the value of x in the Equation

$$35+\sqrt[3]{x-5}=40.$$

By transposition, $\sqrt[3]{x-5}=40-35$.

Adding similar terms, and cubing both sides

$$x-5=125\,;$$

which will give $x=130$.

4. Find the value of x in the Equation

$$1+2\sqrt{x}=\sqrt{4x+21}.$$

By transposition, $2\sqrt{x}=\sqrt{4x+21}-1$.

Squaring both sides,

$$4x=4x+21-2\sqrt{4x+21}+1.$$

Transposing, and adding similar terms,

$$2\sqrt{4x+21}=22.$$

Squaring both sides

$$16x+84=484.$$

From this Equation we shall find $x=25$.

5. Find the value of x in the Equation

$$\sqrt{x-32}=\sqrt{x}-\tfrac{1}{2}\sqrt{32}.$$

Squaring both sides,

$$x-32=x-\sqrt{32x}+8.$$

Transposing, and adding similar terms,

$$\sqrt{32x}=40.$$

Squaring both sides,

$$32x=1600.$$

This Equation will give $x=50$.

6. Find the value of x in the Equation

$$3+\sqrt{x+4}\times\sqrt{x-4}=10.$$

Multiplying together the two radical quantities in this Equation,

$$3+\sqrt{x^2-16}=10.$$

Transposing, and uniting similar terms

$$\sqrt{x^2-16}=7.$$

Squaring both sides,

$$x^2-16=49;$$

which will give $x=\pm\sqrt{65}$.

7. Find the value of x in the Equation

$$a+\sqrt{x-3}\times\sqrt{x+3}=4a.$$

Multiplying together the two radical quantities in this Equation,

$$a+\sqrt{x^2-9}=4a.$$

Transposing, and adding similar terms,

$$\sqrt{x^2-9}=3a.$$

Squaring both sides,

$$x^2-9=9a^2.$$

By transposition, $x^2=9a^2+9$.

Hence, we have $x=\sqrt{9a^2+9}=\pm3\sqrt{a^2+1}$, (237).

8. Find the value of x in the Equation

$$\sqrt{x}+\sqrt{5+x}=\frac{10}{\sqrt{5+x}}.$$

Clearing this Equation of its fraction,

$$\sqrt{5x+x^2}+5+x=10.$$

Transposing, and adding similar terms,

$$\sqrt{5x+x^2}=5-x.$$

Squaring both sides,

$$5x+x^2=25-10x+x^2.$$

Transposing, and adding similar terms,

$$15x=25;\ \text{whence } x=1\tfrac{2}{3}.$$

9. Find the value of x in the Equation

$$\frac{4\sqrt{6x}-9}{4\sqrt{6x}+6}=\frac{\sqrt{6x}-2}{\sqrt{6x}+2}.$$

Clearing this Equation of fractions, by multiplying each numerator into the denominator of the other fraction,

$$24x-\sqrt{6x}-18=24x-2\sqrt{6x}-12.$$

Transposing, and uniting similar terms,

$$\sqrt{6x}=6.$$

Squaring both sides,

$$6x=36;\ \text{whence } x=6.$$

10. Find the value of x in the Equation

$$\frac{\sqrt{x+c}+\sqrt{x-c}}{\sqrt{x+c}-\sqrt{x-c}}=b.$$

Multiplying both terms of the fraction by $\sqrt{x+c}+\sqrt{x-c}$, (243...2)

$$\frac{2x+2\sqrt{x^2-c^2}}{2c}=b.$$

Clearing the Equation of its fraction,

$$2x+2\sqrt{x^2-c^2}=2bc.$$

Transposing, $$2\sqrt{x^2-c^2}=2bc-2x.$$

Squaring both sides,

$$4x^2-4c^2=4b^2c^2-8bcx+4x^2.$$

Transposing, and uniting similar terms,

$$8bcx=4b^2c^2+4c^2.$$

This Equation will give

$$x=\frac{4bc^2+4c^2}{8bc}=\frac{b^2c+c}{2b},\ (86).$$

COMPLETE EQUATIONS

In which the Unknown Quantity is contained in a Surd Expression.

32. Find the value of x in the Equation

$$6+\sqrt{3x+4}=11.$$

Transposing, and uniting similar terms,

$$\sqrt{3x+4}=5.$$

Squaring both sides,

$$3x+4=25;\text{ whence } x=7.$$

33. Find the value of x in the Equation

$$24-\sqrt{9x2^2+}=15.$$

Transposing, and uniting similar terms,

$$-\sqrt{2x^2+9=}-9.$$

Squaring both sides,

$$2x^2+9=81;\text{ whence } x=\pm 6.$$

34. Find the value of x in the Equation

$$20-\sqrt{x^3+40}=4.$$

Transposing, and uniting similar terms,

$$-\sqrt{x^3+40}=-16.$$

Squaring both sides,

$$x^3+40=256.$$

Transposing, and uniting similar terms,

$$x^3=216;\text{ whence } x=6.$$

Then, since this Equation has *three* roots (255), the other two roots will be found by reducing it to a *quadratic* by division, (253).

Dividing both sides of the Equation $x^3-216=0$ by $x-6$, we have

$$x^2+6x+36=0,\text{ or}$$
$$x^2+6x=-36:$$

from which we shall find $x=-3+\sqrt{-27}$, *or* $-3-\sqrt{-27}$.

35. Find the value of x in the Equation

$$x+\sqrt{x}=20.$$

Transposing, $\sqrt{x}=20-x.$

Squaring both sides,

$$x=400-40x+x^2;$$

from which we shall find $x=16$, or 25.

36. Find the value of x in the Equation

$$2\sqrt{x}+\sqrt{x+9}=13.$$

Transposing, $\sqrt{x+9}=13-2\sqrt{x}.$

Squaring both sides,

$$x+9=169-52\sqrt{x}+4x.$$

Transposing, and uniting similar terms,

$$52\sqrt{x}=160+3x.$$

Squaring both sides,

$$2704x=25600+960x+9x^2.$$

This Equation will give $x=16$.

37. Find the value of x in the Equation

$$2\sqrt{x}+\sqrt{2x+1}=\frac{21}{\sqrt{(2x+1)}}.$$

Clearing this Equation of its fraction,

$$2\sqrt{2x^2+x}+2x+1=21.$$

Transposing, and adding similar terms,

$$2\sqrt{2x^2+x}=20-2x.$$

Squaring both sides,

$$8x^2+4x=400-80x+4x^2.$$

This Equation will give $x=4$.

38. Find the value of x in the Equation

$$4\sqrt{x+16}=7\sqrt{x+16}-x-6.$$

Transposing, and uniting similar terms,

$$-3\sqrt{x+16}=-x-6.$$

Squaring both sides,

$$9x+144=x^2+12x+36;$$

from which we shall find $x=9$.

39. Find the value of x in the Equation

$$\frac{x+4}{2}=\sqrt{4+\sqrt{2x^3+x^2}}.$$

Clearing this Equation of its fraction,

$$x+4=2\sqrt{4+\sqrt{2x^3+x^2}}.$$

Squaring both sides

$$x^2+8x+16=16+4\sqrt{2x^3+x^2}.$$

Transposing, and uniting similar terms,

$$-4\sqrt{2x^3+x^2}=-x^2-8x.$$

Squaring both sides,

$$32x^3+16x^2=x^4+16x^3+64x^2.$$

Dividing both sides of this Equation by x^2, we have

$$32x+16=x^2+16x+64;$$

from which we shall find $x=12$, or 4.

40. Find the value of x in the Equation

$$(x-2)^2=\frac{x+\sqrt{(x^2-9)}}{x-\sqrt{(x^2-9)}}.$$

Rationalizing the denominator (243...2), we have

$$(x-2)^2=\frac{(x+\sqrt{(x^2-9)})^2}{9}.$$

Extracting the square root of each side of this Equation,

$$x-2=\frac{x+\sqrt{x^2-9}}{3}.$$

Clearing this Equation of its fraction,

$$3x-6=x+\sqrt{x^2-9}.$$

Transposing, and adding similar terms,

$$-\sqrt{x^2-9}=6-2x.$$

Squaring both sides, we have,

$$x^2-9=36-24x+4x^2;$$

which will give $x=5$, or 3.

41. Find the value of x in the Equation

$$x^2-6x+9=\frac{x+\sqrt{(x^2-16)}}{x-\sqrt{(x^2-16)}}.$$

Rationalizing the denominator (243...2), we shall have

$$x^2-6x+9=\frac{(x+\sqrt{(x^2-16)})^2}{16}.$$

Extracting the square root of each side of this Equation,

$$x-3=\frac{x+\sqrt{x^2-16}}{4}.$$

Clearing this Equation of its fraction,

$$4x-12=x+\sqrt{x^2-16}.$$

Transposing, and adding similar terms,

$$-\sqrt{x^2-16}=12-3x.$$

Squaring both sides,

$$x^2-16=144-72x+9x^2;$$

from which we shall find $x=5$, or 4.

42. Find the value of x in the Equation,

$$\sqrt[3]{x^3+37}\times(x^3+37)^{\frac{2}{3}}=64.$$

This Equation may be put under the form

$$(x^3+37)^{\frac{1}{3}}\times(x^3+37)^{\frac{2}{3}}=64.$$

Multiplying together the two binomials in the first member

$$(x^3+37)=64, \text{ (241...3)}.$$

From this Equation we shall have

$$x^3=27;$$

which will give $x=3$.

The other two values of x (255), will be found by reducing the *cubic* Equation to a *quadratic*, (253).

Dividing both sides of the Equation $x^3-27=0$ by $x-3$,

$$x^2+3x+9=0, \text{ or}$$

$$x^2+3x=-9;$$

from which we shall find $x=-\frac{3}{2}\pm\frac{3}{2}\sqrt{-3}$.

Equations of a Quadratic Form with reference to a Power or Root of the Unknown Quantity.

43. Find the value of x in the Equation

$$x^4-2x^2+6=230.$$

The exponent of x in the first term is the square of its exponent in the second.

Transposing, and uniting similar terms,

$$x^4-2x^2=224.$$

Completing the square, we have

$$x^4-2x^2+1=225.$$

Extracting the square root of each side,

$$x^2-1=\pm 15.$$

This Equation will give

$$x^2=16\,;\ \text{whence, } x=\pm 4.$$

44. Find the value of x in the Equation

$$x^6+20x^3-10=59.$$

Transposing, and uniting similar terms,

$$x^6+20x^3=69.$$

Completing the square, we have

$$x^6+20x^3+100=169.$$

Extracting the square root of each side,

$$x^3+10=\pm 13.$$

This Equation will give

$$x^3=3\,;\ \text{whence } x=\sqrt[3]{3}.$$

45. Find the value of x in the Equation

$$2x^4-x^2+20=23.$$

Transposing, and uniting similar terms,

$$2x^4-x^2=3.$$

Dividing both sides by the coefficient of x^4, we have

$$x^4-\frac{x^2}{2}=\frac{3}{2}.$$

Completing the square, we have

$$x^4-\frac{x^2}{2}+\frac{1}{16}=\frac{3}{2}+\frac{1}{16}=\frac{25}{16}.$$

Extracting the square root of both sides,

$$x^2-\frac{1}{4}=\pm\frac{5}{4}.$$

From this Equation we shall find

$$x^2 = \frac{6}{4}; \text{ hence } x = \sqrt{\tfrac{1}{4}}\,\sqrt{6} = \tfrac{1}{2}\sqrt{6}, \text{ (237).}$$

46. Find the value of x in the Equation

$$3x^{\frac{1}{2}} - 5x^{\frac{1}{4}} = -1\tfrac{1}{3}.$$

Dividing both sides by the coefficient of $x^{\frac{1}{2}}$, we have

$$x^{\frac{1}{2}} - \frac{5x^{\frac{1}{4}}}{3} = -\frac{4}{9}.$$

Completing the square,

$$x^{\frac{1}{2}} - \frac{5x^{\frac{1}{4}}}{3} + \frac{25}{36} = -\frac{4}{9} + \frac{25}{36} = \frac{9}{36}.$$

Extracting the square root of each side,

$$x^{\frac{1}{4}} - \frac{5}{6} = \pm\frac{3}{6}.$$

From this Equation we shall find

$$x^{\frac{1}{4}} = \frac{4}{3}, \text{ or } \frac{1}{3}.$$

By raising each of these values to the 4*th power*, we find

$$x = 3\tfrac{13}{81}, \text{ or } \frac{1}{81}.$$

47. Find the value of x in the Equation

$$6x^{\frac{4}{3}} - 5x^{\frac{8}{3}} + 1184 = 0.$$

Transposing, and changing signs,

$$5x^{\frac{8}{3}} - 6x^{\frac{4}{3}} = 1184.$$

Dividing both sides by the coefficient of $x^{\frac{8}{3}}$, we have

$$x^{\frac{8}{3}} - \frac{6x^{\frac{4}{3}}}{5} = \frac{1184}{5}.$$

Completing the square,

$$x^{\frac{8}{3}}-\frac{6x^{\frac{4}{3}}}{5}+\frac{9}{25}=\frac{1184}{5}+\frac{9}{25}=\frac{5929}{25}.$$

Extracting the square root of each side,

$$x^{\frac{4}{3}}-\frac{3}{5}=\pm\frac{77}{5}.$$

This Equation will give

$$x^{\frac{4}{3}}=16, \text{ or } \sqrt[3]{x^4}=16.$$

Cubing both sides of this Equation,

$$=4096;\ \text{whence } x=8.$$

48. Find the value of x in the Equation

$$(x+12)^{\frac{1}{2}}+(x+12)^{\frac{1}{4}}=6.$$

Representing the binomial $(x+12)$ by y, we have

$$y^{\frac{1}{2}}+y^{\frac{1}{4}}=6.$$

Completing the square, we have

$$y^{\frac{1}{2}}+y^{\frac{1}{4}}+\frac{1}{4}=6+\frac{1}{4}=\frac{25}{4}.$$

Extracting the square root of both sides,

$$y^{\frac{1}{4}}+\frac{1}{2}=\pm\frac{5}{2}.$$

From this Equation we shall find

$$y^{\frac{1}{4}}=2;\ \text{hence } y=16.$$

By restoring the binomial value of y, we have

$$x+12=16;\ \text{whence } x=4.$$

49. Find the value of x in the Equation

$$(2x+6)^{\frac{1}{2}}-6=-(2x+6)^{\frac{1}{4}}$$

Transposing, and changing signs,

$$(2x+6)^{\frac{1}{2}}+(2x+6)^{\frac{1}{4}}=6.$$

Completing the square, we have,

$$(2x+6)^{\frac{1}{2}}+(2x+6)^{\frac{1}{4}}+\frac{1}{4}=6+\frac{1}{4}=\frac{25}{4}.$$

Extracting the square root of each side,

$$(2x+6)^{\frac{1}{4}}+\frac{1}{2}=\pm\frac{5}{2}.$$

From this Equation we shall find

$$(2x+6)^{\frac{1}{4}}=2.$$

Raising both sides of this Equation to the 4th power,

$$2x+6=16.$$

This Equation will give

$$x=5.$$

50. Find the value of x in the Equation

$$x^2+11+\sqrt{x^2+11}=42.$$

This Equation may be put under the form

$$(x^2+11)+(x^2+11)^{\frac{1}{2}}=42.$$

Completing the square,

$$(x^2+11)+(x^2+11)^{\frac{1}{2}}+\frac{1}{4}=42+\frac{1}{4}=\frac{169}{4}.$$

Extracting the square root of each side,

$$(x^2+11)^{\frac{1}{2}}+\frac{1}{2}=\pm\frac{13}{2}.$$

This Equation will give

$$(x^2+11)^{\frac{1}{2}}=6.$$

Squaring both sides of this Equation,

$$x^2+11=36.$$

From this Equation we shall find

$$x=\pm 5.$$

51. Find the value of x in the Equation

$$\frac{1}{(2x-4)^2}=\frac{1}{8}+\frac{2}{(2x-4)^4}.$$

Clearing the Equation of its fractions by multiplying its two members by the *least common multiple* of the denominators, which is evidently $8(2x-4)^4$, we have

$$8(2x-4)^2=(2x-4)^4+16.$$

Transposing, and changing the signs,

$$(2x-4)^4-8(2x-4)^2=-16.$$

Completing the square, we have

$$(2x-4)^4-8(2x-4)^2+16=0.$$

Extracting the square root of each side,

$$(2x-4)^2-4=0.$$

This Equation will give

$$2x-4=\pm 2.$$

From which we shall find

$$x=3, \text{ or } 1.$$

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52. Find the value of x in the Equation

$$4x^4-4x^3+\frac{x}{2}=33.$$

This Equation may be reduced to the form of a *quadratic*, thus:— The first two terms of the *square root* of the first side will be found to be

$2x^2-x$; and the *remainder* will be $-x^2+\frac{x}{2}$,

which may be put under the form

$$-\frac{1}{2}(2x^2-x).$$

Now, the *square* of the root found, and the *remainder*, are together equivalent to the first side of the equation; hence, we shall have

$$(2x^2-x)^2-\frac{1}{2}(2x^2-x)=33.$$

Completing the square,

$$(2x^2-x)^2-\frac{1}{2}(2x^2-x)+\frac{1}{16}=33+\frac{1}{16}=\frac{529}{16}.$$

Extracting the square root of each side,

$$(2x^2-x)-\frac{1}{4}=\pm\frac{23}{4}.$$

This Equation will give

$$2x^2-x=6\text{ ; whence }x=2,\text{ or }-1\tfrac{1}{2}.$$

53. Find the value of x in the Equation

$$x^3-8x^2+19x=12.$$

Transposing the 12, and multiplying by x, we have

$$x^4-8x^3+19x^2-12x=0.$$

We have now a Biquadratic Equation, which may be reduced to the form of a Quadratic by the method just exemplified.

We find the first two terms of the *square root* of the first member, by the common rule.

$$\begin{array}{rl} & x^4-8x^3+19x^2-12x\;(x^2-4x \\ & x^4 \\ \cline{2-2} 2x^2-4x) & -8x^3+19x^2 \\ & -8x^3+16x^2 \\ \cline{2-2} & \qquad\qquad 3x^2-12x. \end{array}$$

This remainder may be resolved into $3(x^2-4x)$, in which the binomial factor is the same as the part of the square root above found.

Then we shall have the Equation

$$(x^2-4x)^2+3(x^2-4x)=0.$$

Completing the square,

$$(x^2-4x)^2+3(x^2-4x)+\frac{9}{4}=\frac{9}{4}.$$

Extracting the square root of both sides,

$$(x^2-4x)+\frac{3}{2}=\pm\frac{3}{2}.$$

From this Equation we shall find

$$x^2-4x=-3,\text{ or }0.$$

The Equation $x^2-4x=-3$ will give $x=3$, or 1.
The Equation $x^2-4x=0$ will give $x=4$, or 0.

54. Find the value of x in the Equation

$$x^4-2x^3+x=5112.$$

The first two terms of the square root of the first member are

$$x^2-x;$$

and the remainder is $-x^2+x$; which may be put under the form

$$-(x^2-x).$$

Hence, we shall have the Equation

$$(x^2-x)^2-(x^2-x)=5112.$$

Completing the square, we have

$$(x^2-x)^2-(x^2-x)+\frac{1}{4}=5112+\frac{1}{4}=\frac{20449}{4}.$$

Extracting the square root of both sides,

$$(x^2-x)-\frac{1}{2}=\pm\frac{143}{2}.$$

From this Equation, we shall have

$$x^2-x=72, \text{ or } -71;$$

$$x^2-x=72 \text{ will give } x=9, \text{ or } -8.$$

55. Find the value of x in the Equation

$$x^4+2x^3-7x^2-8x=-12.$$

The first two terms of the square root of the first member are

$$x^2+x;$$

and the remainder is $-8x^2-8x$; which may be expressed by

$$-8(x^2+x).$$

We shall then have the Equation

$$(x^2+x)^2-8(x^2+x)=-12.$$

Completing the square, we find

$$(x^2+x)^2-8(x^2+x)+16=4.$$

Extracting the square root of each side

$$(x^2+x)-4=\pm 2; \text{ hence}$$

$$x^2+x=6, \text{ or } 2.$$

The Equation $x^2+x=6$ will give $x=2$, or -3.
The Equation $x^2+x=2$ will give $x=1$, or -2.

56. Find the value of x in the Equation

$$x^4-10x^3+35x^2-50x+24=0.$$

Transposing, $x^4-10x^3+35x^2-50x=-24.$

The first two terms of the square root of the first side of the Equation are $x^2-5x;$

and the remainder is $10x^2-50x;$ which may be expressed by

$$10(x^2-5x).$$

Hence, we have the Equation

$$(x^2-5x)+10(x^2-5x)=-24.$$

Completing the square,

$$(x^2-5x)+10(x^2-5x)+25=1.$$

Extracting the square root,

$$x^2-5x+5=\pm 1.$$

From the Equation $x^2-5x+5=+1$, x will be found $=4$, or 1.

From the Equation $x^2-5x+5=-1$, x will be found $=3$, or 2.

57. Find the value of x in the Equation

$$x^4-12x^3+44x^2-48x=9009.$$

The first two terms of the square root of the first number are

$$x^2-6x;$$

and the remainder is $8x^2-48x;$ which may be put under the form

$$8(x^2-6x).$$

Hence, we shall have the Equation

$$(x^2-6x)^2+8(x^2-6x)=9009.$$

Completing the square, we have

$$(x^2-6x)^2+8(x^2-6x)+16=9025.$$

Extracting the square root of each side

$$(x^2-6x)+4=\pm 95.$$

From the Equation $x^2-6x+4=+95$, we shall find $x=13$, or -7.

58. Find the value of x in the Equation

$$x^4-8ax^3+8a^2x^2+32a^3x=s.$$

The first two terms of the square root of the first member of the Equation are $x^2-4ax;$

and the remainder is $-8a^2x^2+32a^3x;$ which may be expressed by

$$-8a^2(x^2-4ax).$$

Hence, we shall have

$$(x^2-4ax)^2-8a^2(x^2-4ax)=s.$$

Completing the square, we find

$$(x^2-4ax)^2-8a^2(x^2-4ax)+16a^4=s+16a^4.$$

Extracting the square root of both sides,

$$(x^2-4ax)-4a^2=\pm\sqrt{s+16a^4}.$$

Transposing $-4a^2$, we have

$$x^2-4ax=4a^2\pm\sqrt{s+16a^4}.$$

Completing the square,

$$x^2-4ax+4a^2=8a^2\pm\sqrt{3+16a^4}.$$

Extracting the square root,

$$x-2a=\pm\sqrt{8a2\pm\sqrt{s+16a^4}}.$$

Transposing $-2a$, we find

$$x=2a\pm\sqrt{8a^2\pm\sqrt{s+16a^4}}.$$

PROBLEMS

In Pure Equations and Affected Quadratics Containing but One Unknown Quantity.

1. Find two numbers such that their product shall be 750, and the quotient of the greater divided by the less, $3\frac{1}{3}$.

Let x represent the greater number; then

$\frac{750}{x}$ represents the less; and

$\frac{x^2}{750}$ represents the *quotient* of the greater divided by the less.

We shall then have the Equation

$$\frac{x^2}{750}=3\frac{1}{3},$$

from which we shall find $x=50$, the greater; then $\frac{750}{50}=15$, is the less.

2. Find a number such that if $\frac{1}{7}$ and $\frac{1}{8}$ of it be multiplied together, and the product divided by 3, the quotient will be $298\frac{2}{3}$.

Let x represent the number; then

$\frac{x^2}{56}$ represents the product of $\frac{1}{7}$ and $\frac{1}{8}$ of the number; and

$\frac{x^2}{168}$ represents this product divided by 3.

We shall then have the Equation

$$\frac{x^2}{168}=298\tfrac{2}{3},$$

from which we shall find $x=224$.

3. A mercer bought a piece of silk for £16 4*s.*; and the number of shillings that he paid per yard, was to the number of yards, as 4 to 9. How many yards did he buy? and what was the price per yard?

Let x represent the number of yards he bought; then

$\frac{4x}{9}$ represents the number of shillings *per yard;* and

$\frac{4x^2}{9}$ represents the number of shillings the *whole* cost; also

£16 4*s.* is equal to 324 shillings.

Hence, we shall have the Equation

$$\frac{4x^2}{9}=324;\ \text{whence } x=27 \text{ yards.}$$

Then $\frac{4}{9}$ of $27=12$ shillings was the price per yard.

4. Find two numbers which shall be to each other as 2 to 3, and the sum of whose squares shall be 208.

Let x represent the greater number; then

$\frac{2x}{3}$ represents the less number; and the Equation will be

$$x^2+\frac{4x^2}{9}=208;$$

which will give $x=12$, the greater number; hence $\frac{2}{3}$ of $12=8$, is the less number.

5. A person bought a quantity of cloth for \$120; and if he had bought 6 yards more for the same sum, the price per yard would have been \$1 less. What was the number of yards? and the price per yard?

Let x represent the number of yards; then

$\frac{120}{x}$ represents the price per yard; and

$\frac{120}{x+6}$ represents what would have been the price per yard, had he bought 6 yards more for the same sum.

Then, by the conditions of the problem,

$$\frac{120}{x+6}=\frac{120}{x}-1;$$

from which x will be found equal to 24, the number of yards; then $\frac{120}{24}=5$ dollars, was the price per yard.

6. Divide the number 20 into two such parts, that the squares of these parts may be in the proportion of 4 to 9.

Let x represent the less part; then

$20-x$ represents the greater.

Hence, by the conditions of the problem,

$$x^2:(20-x)^2::4:9;\text{ or}$$
$$x^2:400-40x+x^2::4:9.$$

This proportion converted into an Equation, gives

$$9x^2=1600-160x+4x^2, (149).$$

From this Equation we shall find $x=8$, the less part; hence $20-8=12$, is the greater part.

7. A merchant bought a quantity of flour for \$100, which he sold again at \$5¼ per barrel, and in so doing gained as much as each barrel cost him. What was the number of barrels?

Let x represent the number of barrels; then

$\frac{100}{x}$ represents the *cost* of each barrel; and

$x\times5\frac{1}{4}$, or $\frac{21x}{4}$, represents what he sold the *whole* for.

Then, by the conditions of the problem,

$$\frac{21x}{4}=100+\frac{100}{x};$$

which will give $x=20$.

8. Divide the number 800 into two such parts that the less divided by the greater, may be to the greater divided by the less as 9 to 25.

Let x represent the less part; then
$800-x$ represents the greater; and we shall have

$$\frac{x}{800-x}:\frac{800-x}{x}::9:25.$$

Converting this proportion into an Equation (149),

$$\frac{25x}{800-x}=\frac{7200-9x}{x}.$$

Clearing this Equation of fractions, &c., we have

$$25x^2=5760000-14400x+9x^2.$$

This Equation will give $x=300$, the less part; hence, $800-300=500$, is the greater part.

9. Two fields which differ in quantity by 10 acres, were each sold for \$2800, and one of them was valued at \$5 an acre more than the other. What was the number of acres in each?

Let x represent the number in the greater; then
$x-10$ represents the number in the less;
$\frac{2800}{x}$ represents the price of the greater, *per acre;* and
$\frac{2800}{x-10}$ represents the price of the less, *per acre.*

Now, it is evident that the *smaller* of the two fields was valued at \$5 an acre more than the *larger;* hence

$$\frac{2800}{x-10}=\frac{2800}{x}+5.$$

Clearing this Equation of fractions,

$$2800x=2800x-28000+5x^2-50x;$$

from which x will be found equal to 80 acres, the number in the greater field; hence $80-10=70$ acres, was the number in the less field.

10. A and B started together on a journey of 150 miles. A traveled 3 miles an hour more than B, and completed the journey $8\frac{1}{3}$ hours before him. At what rate did each travel per hour?

Let x represent the number of miles A traveled per hour; then
$x-3$ represents the number of miles B traveled per hour;
$\frac{150}{x}$ represents the *number of hours* A was traveling; and
$\frac{150}{x-3}$ represents the *number of hours* B was traveling
Then, by the conditions of the problem,

$$\frac{150}{x}=\frac{150}{x-3}-8\tfrac{1}{3}.$$

Clearing this Equation of fractions, &c., we find

$$25x^2-75x=1350.$$

From this Equation we shall find $x=9$ miles, the distance A traveled per hour; hence, $9-3=6$ miles, is the distance B traveled per hour.

11. A man traveled 105 miles, and then found that if he had gone 2 miles less per hour, he would have been 6 hours longer on his journey. At what rate did he travel per hour?

Let x represent the number of miles he went per hour; then
$\frac{105}{x}$ represents the *number of hours* he was traveling; and
$\frac{105}{x-2}$ represents the number of hours he would have traveled, had he gone 2 miles *less* per hour.
Then, by the conditions of the problem,

$$\frac{105}{x-2}=\frac{105}{x}+6.$$

Clearing this Equation of fractions, &c., we have

$$6x^2-12x=210;\ \text{whence}\ x=7\ \text{miles}$$

12. A person has two pieces of silk which together contain 14 yards. Each piece is worth as many shillings per yard as there are yards in the piece, and their whole values are in the proportion of 9 to 16. How many yards are there in each piece?

Let x represent the number in the less piece; then
$14-x$ represents the number in the greater;
$x\times x$, or x^2, represents the *value* of the less piece; and
$(14-x)\times(14-x)$, or $(14-x)^2$, represents the *value* of the greater piece.

Then, by the conditions of the problem,

$$x^2:(14-x)^2::9:16.$$

Converting this proportion into an Equation,

$$16x^2=9(14-x)^2.$$

Developing the second member of this Equation,

$$16x^2=1764-252x+9x^2;$$

which will give $x=6$ yards, the less piece; hence $14-6=8$ yards, is the greater piece.

13. A merchant sold a piece of linen for \$39, and in so doing gained as much per cent as it cost him. What was the cost of the linen?

Let x represent the cost of the linen; then since he gained as much per cent as it cost him,

$\frac{x}{100}$ represents what he gained on one dollar (171); and

$x\times\frac{x}{100}$, or $\frac{x^2}{100}$, represents the *whole* gain, (172).

The Equation will then be

$$x+\frac{x^2}{100}=39;\ \text{whence } x=\$30.$$

14. A grazier bought as many sheep as cost him \$100. After reserving 5 of the number, he sold the remainder for \$135, and gained \$1 a head on them. How many sheep did he buy?

Let x represent the number of sheep; then

$\frac{100}{x}$ represents the *cost per head;*

$x-5$ represents the number of sheep he had after reserving 5; and

$\frac{135}{x-5}$ represents what he sold the remainder for, *per head.*

Then, by the conditions of the problem,

$$\frac{135}{x-5}=\frac{100}{x}+1.$$

Clearing this Equation of fractions, &c., we have

$$x^2-40x=500;$$

which will give $x=50$ sheep.

15. Find two numbers which shall be in the proportion of 7 to 9, and have the difference of their squares equal to 128.

Let x represent the greater; then

$\frac{7x}{9}$ represents the less; and

$x^2 - \frac{49x^2}{81}$ represents the difference of their squares.

The Equation will then be

$$x^2 - \frac{49x^2}{81} = 128;$$

from which we shall find $x = 18$, the greater number; hence $\frac{7}{9}$ of $18 = 14$, is the less number.

16. An officer would arrange 2400 men in a solid body, so that each rank may exceed each file by 43 men. How many must be placed in rank and file?

Let x represent the number in each *rank;* then

$x - 43$ represents the number in each *file.*

Then, since the whole number of men is equal to the number in each *rank* × the number in each *file*, we have

$$x(x-43) = 2400.$$

From this Equation we shall find $x = 75$, the number in each rank; hence $75 - 43 = 32$, is the number in each file.

17. Two partners gained £18 by trade. A's money was employed in the business 12 months; and B's, which was £30, 16 months. A received for his capital and gain £26. What was the amount of his capital?

Let x represent A's capital; then

$26 - x$ represents his *gain;* and

$$18 - (26 - x), \text{ or } x - 8, \text{ is B's gain.}$$

Since the gain is in the *compound ratio* of the *capital* and the *time* it was employed, we have the proportion

$$12x : 30 \times 16 :: 26 - x : x - 8.$$

Converting this proportion into an Equation,

$$12x^2 - 96x = 12480 - 480x;$$

from which x will be found $= 20$.

18. A detachment from an army was marching in regular column, with 5 men more in depth than in front; but upon the enemy's coming in sight, the front was increased by 845 men; and by this movement the detachment was drawn up in five lines. What was the number of men?

Let x represent the number of men in *front;* then
$x+5$ represents the number of men in *depth;* and
$x(x+5)$ represents the *whole number* of men.

Now, $x+845$ represents the number in *front* after it was *increased* by 845 men.

And since the detachment was then drawn up in 5 lines, it had 5 men in depth.

Hence, $5(x+845)$ also represents the *whole number* of men.

We shall therefore have the Equation

$$x(x+5)=5(x+845).$$

Developing both sides of this Equation,

$$x^2+5x=5x+4225;$$

from which we shall find $x=65$, the number in front; hence $65+5=70$, was the number in depth.

Then, $65\times70=4550$ was the whole number of men.

19. A company at a tavern had £8 15*s.* to pay, but before their bill was settled, two of them went away, when those who remained had 10*s.* apiece more to pay than before. How many were there in the company at first?

Let x represent the number; then since
£8 15*s.* is equal to 175 shillings,
$\frac{175}{x}$ represents the number of shillings each one of the *original* company should have paid; and
$\frac{175}{x-2}$ represents the number of shillings each one paid *after two had departed,*

Then, by the conditions of the problem, we shall have

$$\frac{175}{x-2}=\frac{175}{x}+10.$$

Clearing the Equation of fractions, &c., we have

$$10x^2-20x=350;\text{ whence } x=7 \text{ persons.}$$

20. Some gentlemen made an excursion, and each one took the same sum. Each gentleman had as many servants as there were gentlemen, and the number of dollars which each had was double the whole number of servants; also the whole sum taken was $3456. What was the number of gentlemen?

Let x represent the number; then
$x \times x$, or x^2, represents the number of servants;
$2x^2$ represents the number of dollars each gentleman took; and
$2x^2 \times x$, or $2x^3$, represents the *whole sum* taken.
The Equation will then be

$$2x^3 = 3456; \text{ whence } x = 12.$$

21. Divide the number 20 into two such parts, that the product of the whole number and one of the parts shall be equal to the square of the other.

Let x represent the less number; then
$20 - x$ represents the greater.
We shall then have the Equation

$$20x = (20 - x)^2.$$

Developing the second member of this Equation,

$$20x = 400 - 40x + x^2.$$

This Equation will give $x = 30 - 10\sqrt{5}$, the less number; hence $20 - (30 - 10\sqrt{5}) = 10\sqrt{5} - 10$, is the greater.

22. A laborer dug two trenches, one of which was 6 yards longer than the other, for £17 16*s*., and the digging of each cost as many shillings per yard as there were yards in its length. What was the length of each?

Let x represent the number of yards in the first; then
$x + 6$ represents the number of yards in the second;
$x \times x$, or x^2, represents the cost of the first, *in shillings;* and
$(x+6) \times (x+6)$, or $(x+6)^2$, represents the cost of the second, *in shillings*.
Since £17 16*s*. is equal to 356 shillings, the Equation will be

$$x^2 + (x+6)^2 = 356;$$

from which we shall find $x = 10$ yards, the length of the first; hence $10 + 6 = 16$ yards, was the length of the second.

23. There are two numbers whose product is 120, and if 2 be added to the less, and 3 subtracted from the greater, the product of the sum and the remainder will also be 120. What are the two numbers?

Let x represent the less; then

$\frac{120}{x}$ represents the greater.

By the conditions of the problem, the Equation will then be

$$(x+2)\times\left(\frac{120}{x}-3\right)=120.$$

Multiplying together the binomials in the first member,

$$\frac{114x-3x^2+240}{x}=120.$$

From this Equation x will be found equal to 8, the less number; hence $\frac{120}{8}=15$, is the greater number

24. Two persons lay out some money on speculation. A disposes of his bargain for £11, and gains as much per cent as B lays out. B's gain is £36, and it appears that A gains 4 times as much per cent as B. What sum did each lay out?

Let x represent the number of pounds A laid out; then

$11-x$ represents his gain;

$\frac{11-x}{x}$ represents his gain *on one pound;*

$\frac{100(11-x)}{x}$ represents his gain *per cent;* and by the problem

$\frac{100(11-x)}{x}$ represents the number of pounds B laid out.

Now $36\div\frac{100(11-x)}{x}$, or $\frac{9x}{25(11-x)}$, represents B's gain on one dollar; and

$100\times\frac{9x}{25(11-x)}$, or $\frac{36x}{(11-x)}$, represents his gain *per cent.*

By the conditions of the problem, we shall then have

$$\frac{100(11-x)}{x}=4\times\frac{36x}{(11-x)}.$$

Clearing this Equation of fractions,

$$100(11-x)^2=144x^2.$$

Developing the first member of this Equation,

$$12100-2200x+100x^2=144x^2;$$

which will give $x=£5$, the sum A laid out; hence $\frac{100(11-5)}{5}=£120$, is the sum B laid out.

25. A set out from C towards D, and traveled 7 miles an hour. After he had gone 32 miles, B set out from D towards C, and went each hour $\frac{1}{19}$ of the whole distance; and after he had traveled as many hours as he went miles in one hour, he met A. Required the distance between the two places.

Let x represent the number of miles between the places; then

$\frac{x}{19}$ represents the distance B went *per hour;*

$\frac{x}{19}$ represents the *number of hours* he traveled; and

$\frac{x}{19}\times\frac{x}{19}$, or $\frac{x^2}{361}$, represents the *whole distance* he traveled.

Now, as A traveled 7 miles an hour, and had gone 32 miles before B set out from D,

$32+7\times\frac{x}{19}$, or $32+\frac{7x}{19}$, represents the distance A traveled.

Since both together traveled the whole distance between the two places, the Equation will be

$$\frac{x^2}{361}+32+\frac{7x}{19}=x.$$

Clearing this Equation of fractions by multiplying it by the least common multiple of the denominators, which is 361, we have

$$x^2+11552+133x=361x;$$

from which x will be found equal to 152, or 76 miles.

Solution of Two Equations—One or both of the Second or a Higher Degree—Containing Two Unknown Quantities.

1. Find the values of x and y in the Equations

$$2x+y=10,\text{ and }2x^2-xy+3y^2=54.$$

From the first Equation we find

$$x=\frac{10-y}{2}.$$

$$\text{Then }2x^2=\frac{2(10-y)^2}{4},\text{ and }xy=\frac{10y-y^2}{2}$$

Substituting these values of $2x^2$ and xy in the second Equation,

$$\frac{2(10-y)^2}{4}-\frac{10y-y^2}{2}+3y^2=54.$$

Clearing this Equation of fractions,

$$2(10-y)^2-20y+2y^2+12y^2=216.$$

From this Equation we shall find $y=4$, or $-\frac{1}{4}$ (262); and substituting the first of these values for y in the first Equation, we find $x=3$; substituting the second value in the same Equation, we find $x=\frac{41}{8}$.

2. Find the values of x and y in the Equations

$$x+y=9,\text{ and }x^2+y^2=45.$$

From the first Equation we find

$$x=9-y;\text{ hence}$$

$$x^2=81-18y+y^2.$$

Substituting this value of x^2 in the second Equation,

$$81-18y+2y^2=45;$$

which will give $y=6$, or 3; and substituting these values successively, for y in the first Equation, we find $x=3$, or 6.

3. Find the values of x and y in the Equations

$$xy=28,\text{ and }x^2+y^2=65.$$

Dividing the first Equation by y, we have

$$x=\frac{28}{y};\text{ hence}$$

$$x^2=\frac{784}{y^2}.$$

The second Equation gives

$$x^2=65-y^2.$$

Putting these two values of x^2 equal to each other,

$$\frac{784}{y^2}=65-y^2.$$

Clearing this Equation of its fraction,

$$784=65y^2-y^4.$$

From this Equation y will be found $=\pm4$, or ±7, (263). The value of x will then be found to be ±7, or ±4.

4. Find the value of x and y in the Equations

$$x+4y=14,\text{ and }4x-2y+y^2=11.$$

By transposition in the first Equation,

$$x=14-4y.$$

Substituting this value of x in the second Equation,

$$56-18y+y^2=11;\text{ hence}$$

$$y=3,\text{ or }15.$$

We shall now easily find $x=2$, or -46.

5. Find the values of x and y in the Equations

$$x+2y=7,\text{ and }x^2+3xy-y^2=23.$$

Transposing $2y$ in the first Equation,

$$x=7-2y.$$

Hence $x^2=49-28y+4y^2$, and $3xy=21y-6y^2$.

Substituting these values of x^2 and $3xy$ in the second Equation, and adding similar terms in the first member of the resulting Equation,

$$49-7y-3y^2=23;\text{ hence}$$

$$y=2,\text{ or }-4\tfrac{1}{3}.$$

Substituting these values of y, successively, in the first Equation, we shall find $x=3$, or $15\frac{2}{3}$.

6. Find the values of x and y in the Equations

$$x+\frac{y}{2}=11, \text{ and } xy+2y^2=120.$$

Clearing the first Equation of its fraction,

$$2x+y=22\text{ ; hence}$$
$$y=22-2x.$$

Then $xy=22x-2x^2$, and $2y^2=968-176x+8x^2$.

Substituting these values of xy and $2y^2$ in the second Equation, transposing, and uniting similar terms,

$$6x^2-154x=-848\text{ ;}$$

from which x will be found equal to 8, or $17\frac{2}{3}$.

The value of y will then be found to be 6, or $-13\frac{1}{3}$.

7. Find the values of x and y in the Equations

$$x-y=-2, \text{ and } x+\frac{y}{10}=\frac{3xy}{10}.$$

Clearing the second Equation of fractions,

$$10x+y=3xy.$$

Transposing y in the first Equation,

$$x=y-2.$$

Hence $10x=10y-20$, and $3xy=3y^2-6y$.

Substituting these values of $10x$ and $3xy$ in the third Equation, and reducing, we find

$$17y-3y^2=20\text{ ; hence}$$
$$y=4, \text{ or } 1\tfrac{2}{3}.$$

Substituting these values of y, successively in the first Equation, we find $x=2$, or $-\frac{1}{3}$.

8. Find the values of x and y in the Equations

$$x-\frac{x-y}{2}=4, \text{ and } y-\frac{x+3y}{x+2}=1.$$

Clearing these Equations of fractions, and reducing each resulting Equation, we have

$$x+y=8, \text{ and } xy-y-2x=2.$$

Transposing y in the third Equation,

$$x=8-y; \text{ hence}$$

$$xy=8y-y^2.$$

Substituting these values of x and xy in the fourth Equation, and reducing,

$$9y-y^2=18;$$

which will give $y=6$, or 3; and we shall now find $x=2$, or 5.

9. Find the values of x and y in the Equations

$$5(x+y)=13(x-y), \text{ and } x+y^2=52.$$

Developing both sides of the first Equation, and reducing,

$$8x=18y; \text{ hence}$$

$$x=\frac{9y}{4}.$$

From the second Equation, we have

$$x=25-y^2.$$

Equating these two values of x, we have

$$\frac{9y}{4}=25-y^2.$$

From this Equation we shall find $y=4$, or $-6\frac{1}{4}$; and the value of x will then be found to be 9, or $-14\frac{1}{16}$.

Solutions by Means of an Auxiliary Unknown Quantity.

10. Find the values of x and y in the Equations

$$x^2+xy=54, \text{ and } 2xy+y^2=45.$$

Assuming $x=vy$, and substituting this value for x in each of the given Equations, we have

$$v^2y^2+vy^2=54, \text{ and } 2vy^2+y^2=45.$$

From the first of these Equations,

$$y^2=\frac{54}{v^2+v}, \text{ and from the second } y^2=\frac{45}{2v+1}.$$

Equating these two values of y^2, we have

$$\frac{54}{v^2+v}=\frac{45}{2v+1}.$$

Clearing this Equation of fractions,

$$108v+54=45v^2+45v;\text{ hence}$$

$$v=2,\text{ or}-\frac{3}{5}.$$

Substituting these values of v, successively, for v in the fourth Equation, we have

$$4y^2+y^2=45\text{ and }y^2-\frac{6y^2}{5}=45.$$

The first of these Equations will give $y=\pm3$.
The second of the two will give $y=\pm\sqrt{-225}=\pm15\sqrt{-1}$.
Then, since $x=vy$, we have $x=\pm3\times2=\pm6$; or $x=\pm15\sqrt{-1}\times-\frac{3}{5}=\mp9\sqrt{-1}$.

11. Find the values of x and y in the Equations

$$xy=28,\text{ and }x^2+y^2=65.$$

Substituting vy for x in each of these Equations, we have

$$vy^2=28\text{ and }v^2y^2+y^2=65.$$

From the first of these Equations,

$$y^2=\frac{28}{v}.$$

From the second,

$$y^2=\frac{65}{v^2+1}.$$

Equating these two values of y^2,

$$\frac{28}{v}=\frac{65}{v^2+1}.$$

Clearing this Equation of its fractions,

$$28v^2+28=65v;\text{ hence}$$

$$x=\frac{7}{4},\text{ or }\frac{4}{7}.$$

Substituting the first of these values for v in the third Equation, we shall find $y=\pm 4$.

Substituting the second value for v in the same Equation, we shall find $y=\pm 7$.

Then since $x=vy$, we shall have $x=\pm 4\times\frac{7}{4}=\pm 7$; or $x=\pm 7\times\frac{4}{7}=\pm 4$.

12. Find the values of x and y in the Equations

$$x^2+xy=12, \text{ and } xy-2y^2=1.$$

Substituting vy for x in each of these Equations,

$$v^2y^2+vy^2=12, \text{ and } vy^2-2y^2=1.$$

From the first of these Equations, we shall find

$$y^2=\frac{12}{v^2+v}, \text{ and from the second, } y^2=\frac{1}{v-2}.$$

Equating these values of y^2,

$$\frac{12}{v^2+v}=\frac{1}{v-2}.$$

Clearing this Equation of fractions,

$$12v-24=v^2+v;$$

which will give $v=3$, or 8.

Substituting these values, successively, in the fourth Equation we find

$$y=\pm 1, \text{ or } y=\pm\sqrt{\frac{1}{6}}=\pm\frac{1}{\sqrt{6}}, \text{ (209)}.$$

Then $x=\pm 1\times 3=\pm 3$, or $x=\pm\frac{1}{\sqrt{6}}\times 8=\pm\frac{8}{\sqrt{6}}$.

13. Find the values of x and y in the Equations

$$4x^2-2xy=12, \text{ and } 2y^2+3xy=8.$$

Assuming $y=vx$, and substituting this value for x in the given Equations,

$$4x^2-2vx^2=12, \text{ and } 2v^2x^2+3vx^2=8.$$

Finding the values of x^2 from these Equations, and putting them equal to each other,

$$\frac{6}{2-v}=\frac{8}{2v^2+3v}.$$

Clearing this Equation of fractions,

$$12v^2+18v=16-8v;\text{ hence}$$

$$v=\frac{1}{2},\text{ or}-\frac{8}{3}.$$

The value of x will be easily found to be ±2, or $\mp\sqrt{\frac{9}{7}}=\mp3\sqrt{\frac{1}{7}}$.

Since $y=vx$, we have $y=\pm2\times\frac{1}{2}=\pm1$; or $y=\mp3\sqrt{\frac{1}{7}}\times-\frac{8}{3}$ $=\pm8\sqrt{\frac{1}{7}}$.

14. Find the values of x and y in the Equations

$$3y^2-x^2=39,\text{ and }x^2+4xy=256-4y^2.$$

Substituting vy for x in these Equations

$$3y^2-v^2y^2=39,\text{ and }v^2y^2+4vy^2=256-4y^2.$$

Finding the values of y^2 from these Equations, and equating them, we have

$$\frac{39}{3-v^2}=\frac{256}{v^2+4v+4}.$$

Clearing this Equation of fractions,

$$39v^2+156v+156=768-256v^2.$$

From this Equation we shall find $v=\frac{6}{5}$, or $-\frac{102}{59}$.

We can now easily find $y=\pm5$, or ∓59.

Then, $x=\pm5\times\frac{6}{5}=\pm6$; or $x=\mp59\times-\frac{102}{59}=\pm102$.

Solutions by Means of Two Auxiliary Unknown Quantities.

15. Find the values of x and y in the Equations

$$x+y=12,\text{ and }\frac{x^2}{y}+\frac{y^2}{x}=18.$$

Clearing the second Equation of fractions,

$$x^3+y^3=18xy.$$

Assuming $x=v+z$, and $y=v-z$, we shall have

$$(x+y)=(v+z)+(v-z)=2v=12.$$

Hence $v=6$.

Then $x=6+z$, and $y=6-z$.

Substituting these values of x and y in the third Equation, we have

$$(6+z)^3+(6-z)^3=18(6+z)(6-z).$$

Developing both sides of this Equation, and reducing,

$$54z^2=216; \text{ hence } z=\pm 2.$$

Then since $x=6+z$, we have $x=6\pm 2=8$, or 4.
And since $y=6-z$, we have $y=6\mp 2=4$, or 8.

16. Find the values of x and y in the Equations

$$x+y=10, \text{ and } x^3+y^3=280.$$

Assuming $x=v+z$, and $y=v-z$, we have

$$x+y=2v=10.$$

Hence $v=5$.

Then $x=5+z$, and $y=5-z$.

Substituting these values of x and y in the second Equation,

$$(5+z)^3+(5-z)^3=280.$$

Developing the first member of this Equation, and reducing,

$$30z^2=30; \text{ hence } z=\pm 1.$$

Since $x=6+z$, we have $x=5\pm 1=6$, or 4.
Since $y=6-z$, we have $y=5\mp 1=4$, or 6.

17. Find the values of x and y in the Equations

$$x+y=11, \text{ and } x^4+y^4=2657.$$

Substituting $v+z$ for x, and $v-z$ for y in the first Equation, and uniting similar terms, we find

$$2v=11, \text{ or } v=\frac{11}{2}.$$

$$\text{Therefore } x=\frac{11}{2}+z, \text{ and } y=\frac{11}{2}-z.$$

Substituting these values of x and y in the second Equation,

$$\left(\frac{11}{2}+z\right)^4+\left(\frac{11}{2}-z\right)^4=2657.$$

Developing each term of the first member, of this Equation, by the method of the Binomial Theorem, we have

$$\left(\frac{11}{2}+z\right)^4=\left(\frac{11}{2}\right)^4+4\left(\frac{11}{2}\right)^3z+6\left(\frac{11}{2}\right)^2z^2+4\left(\frac{11}{2}\right)z^3+z^4;$$

$$\left(\frac{11}{2}-z\right)^4=\left(\frac{11}{2}\right)^4-4\left(\frac{11}{2}\right)^3z+6\left(\frac{11}{2}\right)^2z^2-4\left(\frac{11}{2}\right)z^3+z^4;$$

Adding these two Equations together, and recurring to the preceding Equation, we find

$$2\left(\frac{11}{2}\right)^4+12\left(\frac{11}{2}\right)^2z^2+2z^4=2657.$$

This Equation, by developing the two binomials in the first member, becomes

$$\frac{29282}{16}+\frac{1452z^2}{4}+2z^4=2657.$$

Clearing this Equation of fractions,

$$29282+5808z^2+32z^4=42512.$$

From this Equation, omitting imaginary roots, we shall have

$$z=\sqrt{\frac{72}{32}}=\sqrt{\frac{9}{4}}=\pm\frac{3}{2}.$$

Then x being equal to $\frac{11}{2}+z$, we have $x=\frac{11}{2}\pm\frac{3}{2}=7$, or 4.

And y being equal to $\frac{11}{2}-z$, we have $y=\frac{11}{2}\mp\frac{3}{2}=4$, or 7.

18. Find the values of x and y in the Equations,

$$x+y=10, \text{ and } x^5+y^5=17050.$$

Substituting $v+z$ for x, and $v-z$ for y in the first Equation, and adding similar terms,

$$2v=10; \text{ or } v=5.$$

Therefore $x=5+z$, and $y=5-z$.

Substituting these values of x and y in the second Equation,

$$(5+z)^5+(5-z)^5=17050.$$

Developing each binomial in the first member of this Equation, we have

$$(5+z)^5=3125+3125z+1250z^2+250z^3+25z^4+z^5.$$

$$(5-z)^5=3125-3125z+1250z^2-250z^3+25z^4-z^5.$$

Adding these Equations together,

$$(5+z)^5+(5-z)^5=6250+2500z^2+50z^4.$$

Hence we shall have

$$6250+2500z^2+50z^4=17050.$$

From this Equation, omitting imaginary values, we shall find $z=\pm 2$. Hence $x=5\pm 2=7$, or 3; and $y=5\mp 2=3$, or 7.

Miscellaneous Solutions and Exercises.

19. Find the values of x and y in the Equations

$$xy=6, \text{ and } x^2+x=18-y^2-y.$$

From the second Equation, we have

$$x^2+y^2+x+y=18.$$

Adding twice the first Equation,

$$x^2+2xy+y^2+x+y=30.$$

This Equation may be expressed by

$$(x+y)^2+(x+y)=30;$$

which is quadratic with reference to $x+y$, and from which we shall find

$$x+y=5; \text{ or } x+y=-6; \text{ hence}$$
$$x=5-y; \text{ or } x=-6-y.$$

Substituting these values of x, successively, in the first Equation, we have

$$5y-y^2=6, \text{ and } -6y-y^2=6.$$

The first of these Equations will give

$y=3$, or 2; and the second will give $y=-3\pm\sqrt{3}$.

Since $x=5-y$; or $x=-6-y$; we have $x=5-3=2$, $x=5-2=3$, or $x=-6-(-3\pm\sqrt{3})=-3\mp\sqrt{3}$.

20. Find the values of x and y in the Equations

$$x+y=6, \text{ and } x^2y^2+4xy=96.$$

The second Equation may be expressed by

$$(xy)^2+4(xy)=96.$$

This Equation is quadratic with reference to xy, and will give

$$xy=8, \text{ or } -12; \text{ hence}$$

$$x=\frac{8}{y}, \text{ or } -\frac{12}{y}.$$

Substituting the first value of x in the first Equation,

$$\frac{8}{y}+y=6; \text{ whence } y=4, \text{ or } 2.$$

Substituting the second value in the same Equation,

$$-\frac{12}{y}+y=6; \text{ whence } y=3\pm\sqrt{21}.$$

By substituting each of these values of y for y in the first Equation we shall find the corresponding values of x to be 2, 4, and $3\mp\sqrt{21}$.

21. Find the values of x and y in the Equations

$$x^{\frac{2}{3}}y^{\frac{3}{2}}=2y^2, \text{ and } 8x^{\frac{1}{3}}-y^{\frac{1}{2}}=14.$$

Dividing the first Equation by $y^{\frac{3}{2}}$, we find

$$x^{\frac{2}{3}}=2y^{\frac{1}{2}}; \text{ whence } y^{\frac{1}{2}}=\frac{x^{\frac{2}{3}}}{2}.$$

Substituting this value of $y^{\frac{1}{2}}$ in the second Equation,

$$8x^{\frac{1}{3}}-\frac{x^{\frac{2}{3}}}{2}=14.$$

Clearing this Equation of its fraction,

$$16x^{\frac{1}{3}}-x^{\frac{2}{3}}=28.$$

From this Equation (263), we shall find

$$x^{\frac{1}{3}}=14, \text{ or } 2; \text{ hence } x=2744, \text{ or } 8.$$

Substituting each of the values of $x^{\frac{1}{3}}$ in the second Equation,

$$112-y^{\frac{1}{2}}=14, \text{ and } 16-y^{\frac{1}{2}}=14.$$

From the first of these Equations,

$$y^{\frac{1}{2}}=98; \text{ hence } y=9604.$$

From the second,

$$y^{\frac{1}{2}}=2;\ \text{hence } y=4.$$

22. Find the values of x and y in the Equations

$$x^2y-xy=6, \text{ and } x^3y-y=21.$$

Dividing the first Equation by the second,

$$\frac{x^2y-xy}{x^3y-y}=\frac{6}{21}.$$

Reducing each member of this Equation to its lowest terms,

$$\frac{x}{x^2+x+1}=\frac{2}{7}.$$

Clearing this Equation of fractions,

$$7x=2x^2+2x+2;$$

from which x will be found $=2$, or $\frac{1}{2}$; and we shall now easily find $y=3$, or -24.

23. Find the values of x and y in the Equations

$$x^3+xy^2=39, \text{ and } x^2y+y^3=26.$$

Dividing the second Equation by the first,

$$\frac{x^2y+y^3}{x^3+xy^2}=\frac{26}{39}.$$

Reducing both members to their lowest terms,

$$\frac{y}{x}=\frac{2}{3};\ \text{whence } y=\frac{2x}{3}.$$

Substituting this value of y in the first Equation,

$$x^3+\frac{4x^3}{9}=39;$$

from which x will be found $=3$; and since $y=\frac{2x}{3}$, we have $y=\frac{2}{3}$ of $3=2$.

24. Find the values of x and y in the Equations

$$x-v=4, \text{ and } x^3-y^33=16.$$

Dividing each side of the second Equation by the corresponding side of the first,

$$x^2+xy+y^2=79.$$

Squaring the first Equation,

$$x^2-2xy+y^2=16.$$

Subtracting this Equation from the one preceding it,

$$3xy=63\text{; whence } x=\frac{21}{y}\cdot$$

Substituting $\frac{21}{y}$ for x in the first Equation,

$$\frac{21}{y}-y=4\text{; hence } y=3\text{, or } -7.$$

The value of x will now be found to be 7, or -3.

25. Find the values of x and y in the Equations

$$x^2-y^2=5\text{, and } x^4-y^4=65.$$

Dividing the second Equation by the first,

$$x^2+y^21=3.$$

Adding this Equation to the first,

$$2x^2=18\text{; then } x=\pm 3.$$

Substituting this value of x in the first Equation, we shall find $y=\pm 2$.

26. Find the values of x and y in the Equations

$$x+y=60\text{, and } 2(x^2+y^2)=5xy.$$

Dividing the second Equation by 2, and transposing

$$x^2+y^2-\frac{5xy}{2}=0.$$

Squaring the first Equation,

$$x^2+y^2+2xy=3600.$$

Subtracting the third Equation from the fourth,

$$\frac{9xy}{2}=3600\text{; hence}$$

$$xy=800\text{, and } x=\frac{800}{y}\cdot$$

Substituting this value of x for x in the first Equation,

$$\frac{800}{y}+y=60;$$

from which y will be found equal to 40, or 20; and x is now easily found equal to 20, or 40.

27. Find the values of x and y in the Equations

$$xy=8, \text{ and } \frac{x^2}{y}+\frac{y^2}{x}=9.$$

Clearing the second Equation of fractions,

$$x^3+y^3=9xy.$$

Substituting for xy in this Equation, its value from the first Equation,

$$x^3+y^3=72.$$

From the first Equation,

$$x=\frac{8}{y}; \text{ then } x^3=\frac{512}{y^3}.$$

Substituting this value of x^3 in the fourth Equation,

$$\frac{512}{y^3}+y^3=72.$$

Clearing this Equation of fractions,

$$512+y^6=72y^3;$$

from which we shall find

$$y^3=64, \text{ or } 8; \text{ then } y=4, \text{ or } 2.$$

We shall now find $x=2$, or 4.

28. Find the values of x and y in the Equations

$$xy=25, \text{ and } x^3+y^3=10xy.$$

From the first Equation,

$$x=\frac{25}{y}; \text{ hence } x^3=\frac{15625}{y^3}.$$

Substituting the values of xy and x^3 in the second Equation,

$$\frac{15625}{y^3}+y^3=250.$$

Clearing this Equation of fractions,

$$15625+y^6=250y^3;$$

from which y^3 will be found $=125$; hence $y=5$.

Substituting 5 for y in the first Equation, we find $x=5$.

29. Find the values of x and y in the Equations

$$x^2-y^2-(x+y)=8, \text{ and } (x-y)^2(x+y)=32.$$

Dividing each Equation by $(x+y)$, we have

$$x-y-1=\frac{8}{x+y};$$

$$(x-y)^2=\frac{32}{x+y}.$$

Transposing -1 in the third Equation, and squaring,

$$(x-y)^2=\left(\frac{8}{x+y}+1\right)^2.$$

Equating this value of $(x-y)^2$, with its value in the fourth Equation,

$$\left(\frac{8}{x+y}+1\right)^2=\frac{32}{x+y}.$$

Developing, and clearing this Equation of fractions, by multiplying it by $(x+y)^2$, we shall find

$$64+16(x+y)+(x+y)^2=32(x+y).$$

Transposing, and uniting similar terms,

$$(x+y)^2-16(x+y)=-64.$$

This Equation will give $x+y=8$; then $x=8-y$.

Substituting this value of x in the first Equation,

$$64-16y-8=8; \text{ hence } y=3.$$

Then since $x=8-y$, we have $x=8-3=5$.

30. Find the values of x and y in the Equations,

$$x+\sqrt{xy}+y=7, \text{ and } x^2+xy+y^2=21.$$

Dividing the second Equation by the first,

$$x-\sqrt{xy}+y=3.$$

Adding this Equation to the first;

$$2x+2y=10;\ \text{then}$$

$$(5)\qquad x+y=5.$$

Subtracting the third Equation from the first,

$$2\sqrt{xy}=4;\ \text{hence}\ x=\frac{4}{y}.$$

Substituting this value of x in the fifth Equation,

$$\frac{4}{y}+y=5;\ \text{whence}\ y=4,\ \text{or}\ 1.$$

Since $x=\frac{4}{y}$, the value of x will be 1, or 4.

31. Find the values of x and y in the Equations

$$\frac{x+y}{2}=\sqrt{xy}+4,\ \text{and}\ \sqrt{xy}=\frac{2xy}{x+y}+\frac{12}{5}.$$

From the first Equation,

$$\sqrt{xy}=\frac{x+y}{2}-4.$$

Clearing this Equation of its fraction,

$$2\sqrt{xy}=x+y-8.$$

Squaring both sides of this Equation,

$$(5)\qquad 4xy=(x+y)^2-16(x+y)+64.$$

Equating the two values of $\sqrt{xy}$ from the second and third Equations,

$$\frac{2xy}{x+y}+\frac{12}{5}=\frac{x+y}{2}-4.$$

Clearing this Equation of fractions, and reducing,

$$20xy=5(x+y)^2-64(x+y).$$

Dividing this Equation by 5, and subtracting the result from the fifth Equation,

$$\frac{16(x+y)}{5}=64.$$

This Equation will give

$$x=20-y.$$

Substituting this value of x in the fifth Equation, and adding similar terms,

$$80y-4y^2=144.$$

From this Equation y will be found equal to 18, or 2.

And since $x=20-y$, we find $x=2$, or 18.

32. Find the values of x and y from the Equation and *proportion*

$$xy^2-x=3;$$

$$x^2y^4-x^2 : x^2+x^2y^2+x^2y^4 :: 5 : 7.$$

In any proportion the difference of the first and second terms is to the first, as the difference of the third and fourth is to the third, (160).

Hence, from the given proportion,

$$2x^2+x^2y^2 : x^2y^4-x^2 :: 2 : 5.$$

Dividing the first antecedent and consequent by x^2 (158),

$$2+y^2 : y^4-1 :: 2 : 5.$$

Converting this proportion into an Equation (149), we have

$$\text{(A)}\quad 10+5y^2=2y^4-2.$$

From the first Equation,

$$y^2=\frac{3+x}{x}.$$

$$\text{Then } 2y^4=\frac{18+12x+2x^2}{x^2}.$$

Substituting these values of y^2 and $2y^4$ in Equation (A), we find

$$10+\frac{15+5x}{x}=\frac{18+12x+2x^2}{x^2}-2.$$

Clearing this Equation of fractions, and reducing,

$$15x^2+3x=18.$$

$$\text{Hence } x=1 \text{ or} -\frac{6}{5}.$$

The value of y will now be found from the first Equation to be 2, or $\sqrt{-\frac{3}{2}}$.

33. Find the values of x and y in the Equations

$$x^2+xy+y^2=13, \text{ and } x^4+x^2y^2+y^4=91.$$

Divididg the second Equation by the first,

$$x^2-xy+y^2=7.$$

Adding this Equation to the first, and dividing the result by 2; and also subtracting the third from the first,

$$x^2+y^2=10.$$
$$2xy=6.$$

Adding these Equations together,

$$x^2+2xy+y^2=16.$$

Subtracting the latter of the same two Equations from the former,

$$x^2-2xy+y^2=4.$$

Extracting the square root of each of these two last Equations we have

$$\text{(A)} \qquad x+y=\pm 4;$$
$$\text{(B)} \qquad x-y=\pm 2.$$

Adding these Equations together,

$$2x=\pm 6; \text{ hence } x=\pm 3.$$

Subtracting Equation (B) from Equation (A), we find

$$2y=\pm 2; \text{ hence } y=\pm 1.$$

34. Find the values of x and y in the Equations

$$x^2y=x^2y^2-x^2, \text{ and } x^2y^2+x^2=x^3y^3-x^3.$$

Dividing each Equation by x^2,

$$y=y^2-1, \text{ and } y^2+1=xy^3-x.$$

From the third Equation we shall find

$$y=\frac{1}{2}\pm\frac{1}{2}\sqrt{5}.$$

$$\text{Hence } y^2=\frac{6}{4}\pm\frac{1}{2}\sqrt{5}, \text{ and } y^3=2\pm\sqrt{5}.$$

From the fourth Equation,

$$x(y^3-1)=y^2+1.$$

Substituting for y^3 and y^2 in this Equation; their values as found above, we find

$$x(2\pm\sqrt{5}-1)=\frac{6}{4}\pm\frac{1}{2}\sqrt{5}+1.$$

Uniting similar terms in both members,

$$x(1\pm\sqrt{5})=\frac{5}{2}\pm\frac{1}{2}\sqrt{5}.$$

Dividing both sides by the coefficient of x,

$$x = \pm \frac{1}{2} \sqrt{5}.$$

35. Find the values of x and y in the Equations

$$xy = x^2 - y^2, \text{ and } x^2 + y^2 = x^3 - y^3.$$

From the first Equation,

$$x^2 - xy = y^2;$$

from which the value of x may be found in terms of y.

Completing the square,

$$x^2 - xy + \frac{y^2}{4} = \frac{5y^2}{4}.$$

Extracting the square root of both sides,

$$x - \frac{y}{2} = \pm\sqrt{\frac{5y^2}{4}} = \pm \frac{y}{2} \sqrt{5}.$$

$$\text{Then } x = \frac{y}{2} \pm \frac{y}{2} \sqrt{5}.$$

Substituting this value of x in the second Equation,

$$\frac{3y^2}{2} \pm \frac{y^2\sqrt{5}}{2} + y^2 = 2y^3 \pm y^3\sqrt{5} - y^3.$$

Clearing this Equation of fractions, and adding similar terms in each member of the resulting Equation,

$$5y^2 \pm y^2\sqrt{5} = 2y^3 \pm 2y^3\sqrt{5}.$$

Dividing both sides by y^2, we find

$$5 \pm \sqrt{5} = 2y \pm 2y\sqrt{5}.$$

This Equation may be expressed by

$$2y(1 \pm \sqrt{5}) = \pm\sqrt{5} + 5.$$

Dividing by $(1 \pm \sqrt{5})$, we have

$$2y = \pm\sqrt{5}; \text{ hence } y = \pm \frac{1}{2} \sqrt{5}.$$

Substituting this value of y in the first Equation,

$$\pm \frac{x\sqrt{5}}{2} = x^2 - \frac{5}{4}; \text{ or}$$

$$x^2 \mp \frac{x\sqrt{5}}{2} = \frac{5}{4}.$$

In this Equation, *half the coefficient* of x is $\frac{\sqrt{5}}{4}$, and the *square* is therefore $\frac{5}{16}$.

Completing the square of the preceding Equation,

$$x^2 \mp \frac{x\sqrt{5}}{2} + \frac{5}{16} = \frac{25}{16}.$$

Extracting the square root of both sides,

$$x \mp \frac{\sqrt{5}}{4} = \pm \frac{5}{4}.$$

$$\text{Then } x = \pm \frac{5}{4} \pm \frac{\sqrt{5}}{4} = \frac{1}{4}(\pm 5 \pm \sqrt{5}).$$

36. Find the values of x and y in the Equations

$$x^3 - x^2y - xy^2 + y^3 = 576, \text{ and } x^3 + x^2y + xy^2 + y^3 = 2336.$$

Subtracting the first Equation from the second,

$$2x^2y + 2xy^2 = 1760.$$

Adding this Equation to the second,

$$x^3 + 3x^2y + 3xy^2 + y^3 = 4096.$$

Extracting the cube root of both sides,

$$\text{(A)} \qquad x + y = 16.$$

Dividing the second Equation by Equation (A), we have

$$\text{(B)} \qquad x^2 + y^2 = 146.$$

From Equation (A) we find

$$x = 16 - y.$$

Squaring this value of x and substituting the result for x^2 in Equation (B), we find

$$256 - 32y + 2y^2 = 146;$$

from which y will be found $= 11$, or 5; and since $x = 16 - y$, we find $x = 5$, or 11.

37. Find the values of x and y in the Equations

$$xy = 320, \text{ and } x^3 - y^3 = 61(x - y)^3.$$

Dividing the second Equation by $(x-y)$,

$$x^2+xy+y^2=61\,(x-y)^2.$$

Developing the second member of this Equation,

$$x^2+xy+y^2=61x^2-122xy+61y^2.$$

Substituting 320 for xy, and reducing,

$$60x^2+60y^2=39360.$$

Dividing both sides by 60,

$$\text{(A)} \qquad x^2+y^2=656.$$

From the first Equation,

$$x=\frac{320}{y}\,;\ \text{then } x^2=\frac{102400}{y^2}.$$

Substituting this value of x^2 in Equation (A), we have

$$\frac{102400}{y^2}+y^2=656\,;$$

from which y will be found equal to 20, or 16.

Since $x=\dfrac{320}{y}$, we find $x=16$, or 20.

38. Find the values of x and y in the Equations

$$x^2+y^2+xy=37\,;\ x^2+z^2+xz=49\,;\ y^2+z^2+yz=61.$$

Subtracting the first Equation from the second,

$$z^2-y^2+xz-xy=12.$$

Decomposing the first member of this Equation,

$$(z+y)(z-y)+x(z-y)=12\,;\ \text{or}$$
$$(z+y+x)(z-y)=12.$$

$$\text{Hence } z+y+x=\frac{12}{z-y}.$$

Subtracting the second Equation from the third,

$$y^2-x^2+yz-xz=12.$$

Decomposing the first member,

$$(y+x)(y-x)+z(y-x)=12\,;\ \text{or}$$
$$(y+x+z)(y-x)=12\,;\ \text{hence}$$

$$\text{(A)} \qquad y+x+z=\frac{12}{y-x}.$$

Equating the two values found for the trinomial $(z+y+x)$, we have

$$\frac{12}{z-y}=\frac{12}{y-x};\text{ then}$$

$$z-y=y-x;\text{ hence}$$

$$2y=x+z.$$

Substituting $2y$ for $x+z$ in Equation (A), and reducing,

$$y^2-yx=4;$$

from which we shall find $x=\dfrac{y^2-4}{y}$.

Substituting this value of x in the first Equation,

$$\frac{y^4-8y^2+16}{y^2}+2y^2-4=37.$$

Reducing this Equation, we find

$$3y^4-49y^2=-16.$$

From this Equation, we shall find $y=4$.

Since $x=\dfrac{y^2-4}{y}$, we find $=x3$; and z is now easily found equal to 5.

PROBLEMS

In Quadratic Equations of one or more Unknown Quantities.

1. The *area* of a rectangular lot of ground is 384 square rods, and its length is to its breadth as 3 is to 2. Required the length and breadth of the lot.

The area, in square measure, of a rectangle, is expressed by the product of the number of *linear* units in its *length* × the number of linear units in its *breadth.*

The length and breadth must be taken in the same denomination in multiplying: the *area* will be found in the corresponding denomination of *square measure.*

Let x represent the length, and y the breadth of that lot.

By the conditions of the problem, we shall then have

$$xy=384;$$

$$x:y::3:2.$$

Converting this proportion into an Equation (149), we find

$$2x=3y;$$

from which and the first Equation, x and y will be found equal to 24 and 16 rods, respectively.

2. The length of a rectangular garden exceeds its breadth by 6 rods, and its area is 216 square rods. What are the length and breadth of the garden?

Let x represent the length, and y the breadth; then

$$x=y+6;$$
$$xy=216;$$

which will give $x=18$, and $y=12$ rods.

3. Find two numbers whose sum shall be 24, and whose product shall be equal to 35 times their difference.

Let x represent the greater, and y the less number; then

$$x+y=24; \text{ and}$$
$$xy=35(x-y).$$

From the first Equation,

$$x=24-y, \text{ and from the second } x=\frac{35y}{35-y}$$

Equating two values of x, we have

$$24-y=\frac{35y}{35-y};$$

which may be reduced to

$$y^2-94y=-840;$$

from which y will be found $=10$; and by substituting this value of y in the first Equation, x is found $=14$.

4. Divide a line 20 inches in length into two such parts that the rectangle or product of the whole line and one of the parts shall be equal to the square of the other part.

Let x represent the shorter, and y the longer part.

The Equations will then be

$$x+y=20;$$

$$20x=y^2.$$

These Equations will give $x=30-10\sqrt{5}$, and $y=10\sqrt{5}-10$.

5. Find the dimensions of a rectangular field, so that its length shall be equal to twice its breadth, and its area 800 square rods.

Let x represent its length, and y its breadth; then

$$x=2y; \text{ and}$$

$$xy=800;$$

from which we shall find $x=40$, and $y=20$ rods.

6. The sum of the two digits of a certain number is 10, and if their product be increased by 40, the digits will be reversed. What is the number?

Let x represent the *tens'* figure, and y the *units'* figure of the number; then

$10y+x$ represents the number with its digits *reversed.*

We shall, therefore, have the following Equations:

$$x+y=10;$$

$$xy+40=10y+x.$$

From the first Equation,

$$x=10-y.$$

Substituting this value of x in the second Equation, and reducing,

$$y^2-y=30; \text{ hence}$$

$$y=6; \text{ and } x \text{ is easily found } =4.$$

Therefore, the number is 46.

7. The sum of two fractions is $1\frac{1}{6}$, and the sum of their *reciprocals* is $3\frac{1}{2}$. What are the two fractions?

Let x and y represent the fractions; then

$$x+y=1\tfrac{1}{6}; \text{ and}$$

$$\frac{1}{x}+\frac{1}{y}=3\tfrac{1}{2}.$$

Clearing this Equation of fractions, we have

$$6x+6y=7\text{; and}$$
$$2y+2x=7xy.$$

From the first of these Equations, we find

$$x=\frac{7-6y}{6};$$

and substituting this value in the second,

$$2y+\frac{14-12y}{6}=\frac{49y-42y^2}{6}.$$

Clearing this Equation of fractions,

$$12y+14-12y=49y-42y^2;$$
$$\text{or, } 42y^2-49y=-14;$$
$$\text{or, } 6y^2-7y=-2;$$

which gives $y=\frac{2}{3}$. Then $x=1\frac{1}{6}-\frac{2}{3}=\frac{1}{2}$.

8. The *perimeter*, or sum of the four sides of a rectangle, is 112 rods, and its area is 720 square rods. What are the length and breadth of the rectangle?

Let x represent its length, and y its breadth; then

$$2x+2y=112;$$
$$xy=720.$$

These two Equations will give $x=36$, and $y=20$ rods.

9. Divide the number 60 into two such parts that their product shall be to the sum of their squares as 2 to 5.

Let x represent the less, and y the greater part; then

$$x+y=60\text{; and}$$
$$xy : x^2+y^2 :: 2 : 5.$$

Converting this proportion into an Equation,

$$5xy=2x^2+2y^2\text{; or}$$
$$(3) \qquad 2x^2+2y^2-5xy=0.$$

Squaring the first Equation, and multiplying the result by 2, we have

$$(4) \qquad 2x^2+2y^2+4xy=7200.$$

Subtracting the third Equation from the fourth,

$$9xy=7200\text{; hence } x=\frac{800}{y}.$$

Substituting this value of x in the first Equation, we shall find $y=40$; then $x=60-40=20$.

10. A merchant bought a piece of cloth for $120, and after cutting off 4 yards, sold the remainder for what the whole cost him—by which he made $1 a yard on what he sold. How many yards did the piece contain?

Let x represent the number of yards; then

$\frac{120}{x}$ represents what the merchant *paid* per yard;

$\frac{120}{x-4}$ represents what he *received* per yard.

By the conditions of the problem, we then have

$$\frac{120}{x-4}=\frac{120}{x}+1\text{; hence}$$

$$x=24 \text{ yards.}$$

11. Divide the number 100 into two such parts that the difference of their square roots shall be 2.

Let x represent the greater, and y the less part; then

$$x+y=100\text{; and}$$
$$\sqrt{x}-\sqrt{y}=2.$$

Transposing $\sqrt{y}$ to the second member, and squaring both sides,

$$x=4+4\sqrt{y}+y.$$

Substituting this value of x in the first Equation,

$$4+4\sqrt{y}+2y=100.$$

This Equation may be reduced to

$$2\sqrt{y}+y=48.$$

Transposing y, and squaring both sides,

$$4y=2304-96y+y^2.$$

This Equation will give $y=36$; hence $100-36=64$, is the value of x.

12. A garden which is 20 rods square, is surrounded by a walk whose area is equal to $\frac{1}{4}$ of the area of the garden itself. What is the breadth of the walk?

Let x represent the breadth; then

$2x+20$ rods is the length of the walk on each of two *opposite* sides of the garden;

20 rods is the length on each of the other two sides;

$(2x+40)2$ is the *whole* length of the walk; and

$(2x+40)2x$ is the *area* of the walk.

Since $\frac{1}{4}$ of 20×20, or 100 square rods, is $\frac{1}{4}$ of the area of the garden, the Equation will be

$$(2x+40)2x=100;\ \text{or}$$

$$4x^2+80x=100;$$

from which $x=\sqrt{125}-10=5\sqrt{5}-10$.

13. The sum of the squares of two numbers is 325, and the difference of their squares is 125. What are the numbers?

Let x represent the greater, and y the less number.

The Equations will then be

$$x^2+y^2=325;$$

$$x^2-y^2=125;$$

from which we shall find $x=15$, and $y=10$.

14. The area of a rectangular court-yard, is 875 square rods, and if its length and breadth were each increased by 5 rods, its area would then be 1200 square rods. What are the dimensions of the yard?

Let x represent the length, and y the breadth; then by the conditions of the problem,

$$xy=875;$$

$$(x+5)(y+5)=1200.$$

Developing the first member of the second Equation,

$$xy+5x+5y+25=1200.$$

Substituting the value of xy from the first Equation, for xy in the third Equation, and reducing,

$$x+y=60;\ \text{hence}$$

$$x=60-y.$$

Substituting this value of x in the first Equation,

$$60y-y^2=875;$$

which will give $y=25$ rods; and $60-25=35$ rods, is the value of x.

15. The difference of two numbers is 4, and the difference of their cubes is 448. What are the two numbers?

Let x represent the greater, and y the less number; then

$$x-y=4;$$
$$x^3-y^3=448.$$

Dividing the second Equation by the first,

$$x^2+xy+y^2=112.$$

Squaring the first Equation,

$$x^2-2xy+y^2=16.$$

Subtracting this Equation from the one preceding it,

$$3xy=96; \text{ hence } x=\frac{32}{y}.$$

Substituting this value of x in the first Equation,

$$\frac{32}{y}-y=4.$$

From this Equation, $y=4$; and $x=\frac{32}{4}=8$.

16. A grocer sold 80 pounds of mace and 100 pounds of cloves for £65, and finds that he has sold 60 more of cloves for £20 than of mace for £10. What was the price of each per pound?

Let x represent the price of the mace, and y the price of the cloves per pound; then

$80x$ represents the *whole* value of the mace;
$100y$ represents the whole value of the cloves.

The first Equation will then be

$$80x+100y=65.$$

Now, $\frac{20}{y}$ represents the number of pounds of cloves sold for £20; and

$\frac{10}{x}$ represents the number of pounds of mace sold for £10.

The second Equation will then be

$$\frac{20}{y}-\frac{10}{x}=60.$$

Clearing this Equation of fractions, and dividing the result by 10, we have

$$2x-y=6xy;\text{ hence}$$

$$x=\frac{y}{2-6y}.$$

Substituting this value of x in the first Equation,

$$\frac{80y}{2-6y}+100y=65.$$

Clearing this Equation of fractions, and reducing,

$$60y^2-67y=-13;$$

from which $y=£\frac{1}{4}=5$ shillings.

Substituting $\frac{1}{4}$ for y in the first Equation, we find $x=£\frac{1}{2}=10$ shillings.

17. The fore-wheel of a carriage makes 6 revolutions more than the hind wheel in going 120 yards; but if the circumference of each be increased 1 yard, it will make only four revolutions more in going the same distance. What is the circumference of each wheel?

Let x represent the number of yards in the circumference of the fore-wheel; and y the number in the circumference of the hind wheel; then

$\frac{120}{x}$ represents the number of revolutions made by the fore-wheel in going 120 yards;

$\frac{120}{y}$ represents the number of revolutions made by the hind wheel in going the same distance.

The first Equation will therefore be

$$\frac{120}{x}=\frac{120}{y}+6.$$

In a similar manner, we have for the second Equation

$$\frac{120}{x+1}=\frac{120}{y+1}+4.$$

Clearing each of these Equations of fractions, and reducing,

$$20y=20x+xy;$$

$$29y=xy+31x+1.$$

From the third Equation we find

$$y=\frac{20x}{20-x}.$$

From the fourth Equation, we find

$$y=\frac{31x+1}{29-x}.$$

Equating these two values of y, we have

$$\frac{20x}{20-x}=\frac{31x+1}{29-x}.$$

Clearing this Equation of fractions, transposing, &c.,

$$11x^2-39x=20;\ \text{hence } x=4 \text{ yards.}$$

We shall now easily find $y=5$ yards.

18. Find four numbers in Arithmetical progression, such that the product of the two *extremes* shall be 45, and the product of the two *means* 77.

Let x represent the first term, and y the common difference, of the series of numbers.

Then the numbers will, be

$$x,\ x+y,\ x+2y,\ \text{and } x+3y,\ (175).$$

By the conditions of the problem, we shall then have

$$x^2+3xy=45;$$

$$x^2+3xy+2y^2=77,$$

Subtracting the first Equation from the second,

$$2y^2=32;\ \text{hence } y=4.$$

Substituting 4 for y in the first Equation, we find $x=3$.

The numbers then are

$$3,\ 3+4=7,\ 7+4=11,\ \text{and } 11+4=15.$$

19. A farmer has a field 16 rods long and 12 rods wide, which he wishes to enlarge so that it may contain just twice as much area, without altering the proportion of the sides. What will be the dimensions of the field when thus enlarged?

Let x represent the length, and y the breadth; then since $16\times12\times2=384$ square rods is *twice* the area of the present field, the first Equation will be

$$xy=384.$$

And since its length is $\frac{16}{12}$, or $\frac{4}{3}$ of its breadth, the second Equation will be

$$x=\frac{4y}{3}.$$

This value of x substituted in the first Equation gives

$$\frac{4y^2}{3}=384;\ \text{then}\ y^2=288.$$

$$\text{Hence}\ y=\sqrt{288}=12\sqrt{2},\ (237).$$
$$\text{Then}\ x=\tfrac{4}{3}\ \text{of}\ 12\sqrt{2}=16\sqrt{2}.$$

20. Find three increasing numbers such, that the difference of the first and second shall be two less than the difference of the second and third, their sum 17, and the sum of their squares 115.

Let x represent the *second* number, and y the difference between the first and second: then, by the problem, $y+2$ represents the difference of the second and third.

Hence, $x-y$ represents the first number;
$x+y+2$ represents the third number; and
$3x+2$ represents the sum of all the numbers.

We shall then have the Equations

$$3x+2=17;$$
$$x^2+(x-y)^2+(x+y+2)^2=115.$$

From the first Equation, x will be found $=5$, the second number.

Substituting this value of x in the second Equation,

$$25+(5-y)^2+(7+y)^2=115.$$

Developing both sides of this Equation, and reducing,

$$y^2+2y=8.$$

Hence $y=2$, the second number.

The first number will then be $5-2=3$; and the third number will be $5+2+2=9$.

21. There are two square gardens which together contain 1025 square rods, and a side of the one exceeds a side of the other by 5 rods. What are the sides of the two gardens?

Let x represent a side of the greater, and y a side of the less garden; then

x^2 and y^2 represent the respective areas of the two gardens.

The Equations then are

$$x^2+y^2=1025;$$
$$x-y=5.$$

From these Equations, x will be found $=25$ rods; and $y=20$ rods.

22. Find two numbers such, that their sum, their product, and the difference of their squares shall all be equal to one another.

Let $x+y$ represent the greater number, and $x-y$ the less; then

$2x$ represents their sum;
x^2-y^2 represents their product; and
$4xy$ represents the difference of their squares.

By the conditions of the problem, the Equations are

$$2x=x^2-y^2;$$
$$2x=4xy.$$

From the second Equation,

$$y=\frac{1}{2}.$$

Substituting this value in the first Equation,

$$2x=x^2-\frac{1}{4}.$$

$$\text{Hence,}\quad x=1\pm\sqrt{\frac{5}{4}}.$$

Then $x+y=\frac{3}{2}\pm\sqrt{\frac{5}{4}}$, the greater number.

And $x-y=\frac{1}{2}\pm\sqrt{\frac{5}{4}}$, the less number.

23. A merchant received \$12 for a quantity of linen, and an equal sum, at 50 cents less per yard, for a quantity of calico, which exceeded the quantity of linen by 32 yards. What was the quantity of each?

Let x represent the number of yards of linen, and y the number of yards of calico; then

$\frac{12}{x}$ represents the price of the linen per yard;

$\frac{12}{y}$ represents the price of the calico per yard.

Since 50 cents is equal to $\frac{1}{2}$ of a dollar, by the conditions of the problem, we shall have

$$\frac{12}{y}=\frac{12}{x}-\frac{1}{2}.$$

The second Equation will be

$$y=x+32.$$

From these Equations, $x=16$, and $y=48$ yards.

24. Find two numbers whose sum multiplied by the greater shall be equal to 192, and whose difference multiplied by the less shall be equal to 32.

Let x represent the greater, and xy the less number; then

$$x(x+xy)=192;$$
$$xy(x-xy)=32.$$

The first Equation may be put under the form

$$x^2(1+y)=192; \text{ hence}$$

$$x^2=\frac{192}{1+y}.$$

Developing the first member of the second Equation,

$$x^2y-x^2y^2=32.$$

Substituting for x^2 in this Equation, its value as found above,

$$\frac{192y}{1+y}-\frac{192y^2}{1+y}=32.$$

Clearing this Equation of fractions by multiplying it by the least common multiple of the denominators,

$$192y-192y^2=32+32y; \text{ hence } y=\frac{1}{3}.$$

Since $x^2=\frac{192}{1+y}$, we have $x^2=192\div 1\frac{1}{3}=144$; hence $x=12$.

Since the less number is represented by xy, we have for its value, $12\times\frac{1}{3}=4$.

25. Three merchants gained \$1444; of which their respective shares were such that B's added to the square root of A's, made \$920; but if added to the square root of C's, it made \$912. What was the share of each?

Let x represent A's share, and y B's share; then
$1444-x-y$ represents C's share.
We shall then have the following Equations:

$$y+\sqrt{x}=920;$$

$$y+\sqrt{1444-x-y}=912.$$

Transposing y in the first Equation, and squaring,

$$x=846400-1840y+y^2.$$

Proceeding in a similar manner with the second Equation,

$$1444-x-y=831744-1824y+y^2.$$

Adding together the last two Equations,

$$1444-y=1678144-3664y+2y^2.$$

This Equation will give $y=\$900$; and substituting this value of y for y in the third Equation, we find $x=\$400$.

Hence, C's share was $1444-900-400=\$144$.

26. The sum of three numbers in harmonical progression is 13, and the product of the two extremes is 18. What are the numbers?

Let x and y, respectively, represent the two *extremes*; then
$\dfrac{2xy}{x+y}$ represents the mean term, (184).
The Equations will then be

$$x+y+\frac{2xy}{x+y}=13;$$

$$xy=18.$$

Clearing the first Equation of fractions,

$$(x+y)^2+2xy=13(x+y).$$

Substituting for xy in this Equation its value from the second Eq'n,

$$(x+y)^2+36=13(x+y).$$

This Equation is quadratic with reference to $x+y$, and will give

$$x+y=9;\ \text{hence}\ x=9-y.$$

Substituting this value of x in the second Equation, we find $y=3$; then $x=18\div3=6$; and $\dfrac{2\times6\times3}{6+3}=4$, is the value of the mean term.

27. There is a rectangular field whose length is to its breadth as 4 to 3. A part of this field, which is equal to $\frac{1}{4}$ of the whole, being in meadow, there remain for ploughing 1296 square rods. What are the dimensions of the field?

Let x represent the length, and y the breadth; then
xy represents the area of the field.
We shall then have for the first Equation

$$x = \frac{4y}{3}.$$

And since $\frac{1}{4}$ is in meadow, there remains $\frac{3}{4}$ for ploughing; then

$$1296 = \frac{3xy}{4};$$

from which, and the first Equation, we shall find $x=48$ rods, and $y=36$ rods.

28. The sum of three numbers in geometrical progression is 21, and the sum of their squares 189. What are the numbers?

Let x and y represent the extremes; then
$\sqrt{xy}$ represents the mean term, (189).
By the conditions of the problem,

$$x+\sqrt{xy}+y=21;$$
$$x^2+xy+y^2=189.$$

Dividing the second Equation by the first,

$$x-\sqrt{xy}+y=9.$$

Subtracting this Equation from the first,

$$2\sqrt{xy}=12; \text{ hence } \sqrt{xy}=6, \text{ and } x=\frac{36}{y}.$$

Substituting these values of x and $\sqrt{xy}$ in the first Equation,

$$\frac{36}{y}+6+y=21.$$

From this Equation, $y=12$; hence $\frac{36}{12}=3$, the value of x.

Then. $\sqrt{3\times 12}=6$, the mean term.

29. A and B set out from two places which are distant 110 miles, and traveled towards each other. A went 5 miles an hour; and the number of hours in which they met was greater, by four, than the number of miles C went per hour. What was B's rate of traveling?

Let x represent the number of miles B went per hour; then
$x+4$ represents the number of hours each was traveling;
$5(x+4)$ represents the number of miles A traveled; and
$x(x+4)$ represents the number of miles B traveled.
The Equation then is

$$5(x+4)+x(x+4)=110\text{; whence } x=6 \text{ miles.}$$

30. The arithmetical mean between two numbers exceeds the geometrical mean by 13, and the geometrical mean exceeds the harmonical mean by 12. What are the numbers?

Let x and y represent the numbers; then
$\frac{x+y}{2}$ represents the arithmetical mean, (179);
$\sqrt{xy}$ represents the geometrical mean, (189); and
$\frac{2xy}{x+y}$ represents the harmonical mean, (184).
By the conditions of the problem,

$$\frac{x+y}{2}=\sqrt{xy}+13;$$

$$\sqrt{xy}=\frac{2xy}{x+y}+12.$$

By transposition in the first Equation,

$$\sqrt{xy}=\frac{x+y}{2}-13.$$

Clearing this Equation of fractions, and squaring both sides,

$$4xy=(x+y)^2-52(x+y)+676.$$

Equating the two values of $\sqrt{xy}$ from the second and third Equations,

$$\frac{2xy}{x+y}+12=\frac{x+y}{2}-13.$$

Clearing this Equation of fractions, transposing, &c.,

$$4xy=(x+y)^2-50(x+y).$$

Subtracting this Equation from the fourth, and transposing,

$$2(x+y)=676\text{; hence}$$

$$(8) \qquad x+y=338.$$

Substituting this value of $x+y$ in the third Equation, and uniting similar terms,

$$\sqrt{xy}=156\text{; whence } x=\frac{24336}{y}.$$

Substituting this value of x in the eighth Equation,

$$\frac{24336}{y}+y=338\text{; whence } y=104.$$

$$\text{Since } x=\frac{24336}{y}\text{, we have } x=234.$$

31. Three merchants made a joint stock, by which they gained a sum less than that stock by \$80. A's share of the gain was \$60, and his contribution to the stock was \$17 more than B's; also B and C together contributed \$325. How much did each contribute?

Let x represent A's contribution, y B's, and z C's; then $(x+y+z)-80$ represents their gain.

Now, the *whole stock* is to A's contribution, as the *whole gain* to A's gain.

$$\text{Therefore, } x+y+z : x :: (x+y+z-80) : 60.$$

From the conditions of the problem, we shall also have

$$x-y=17;$$

$$y+z=325.$$

Converting the proportion into an Equation,

$$60(x+y+z)=x(x+y+z-80).$$

Substituting for $y+z$ in this Equation its value from the second Equation,

$$60(x+325)=x(x+245).$$

This Equation will give $x=\$75$; and we shall now, from the first Equation, easily find $y=\$58$; and substituting this value in the second Equation, we find $z=\$267$.

32. Of three numbers in geometrical progression, the greatest exceeds the least by 15, and the difference of the squares of the greatest and the least is to the sum of the squares of the three numbers as 5 to 7. What are the numbers?

Let x represent the least term, and y the ratio of the progression; then

xy represents the mean term; and

xy^2 represents the greatest term, (185).

Then we shall have the following Equations:

$$xy^2-x=15;$$

$$x^2y^4-x^2=\frac{5(x^2+x^2y^2+x^2y^4)}{7}.$$

Dividing the second Equation by x^2, we find

$$y^4-1=\frac{5(1+y^2+y^4)}{7}.$$

This may easily be reduced to

$$2y^4-5y^2=12.$$

From this Equation, $y=2$; and substituting this value for y in the first Equation, we find $x=5$, the least term.

Then $xy=5\times2=10$, the mean term; and $xy^2=5\times4=20$, the greatest term.

33. Two persons set out from different places and traveled towards each other. On meeting, it appeared that A had traveled 24 miles more than B, and that A could have gone B's journey in 8 days, while B would have been 18 days in performing A's journey. What distance was traveled by each?

Let x represent the number of miles A traveled, and y the number of miles B traveled.

The first Equation will then be

$$x=y+24.$$

Now, since A could have gone B's journey in 8 days,

$\frac{y}{8}$ represents the number of miles A went per day.

And since B could have gone A's journey in 18 days,

$\frac{x}{18}$ represents the number of miles B went per day.

Then $x\div\frac{y}{8}$, or $\frac{8x}{y}$, represents the number of days A traveled; and

$y\div\frac{x}{18}$, or $\frac{18y}{x}$, represents the number of days B traveled.

Since both traveled the same number of days, we have

$$\frac{8x}{y}=\frac{18y}{x}.$$

The two Equations will give $x=72$, and $y=48$ miles.

34. The joint stock of two partners was \$416. A's money was in the business 9 months and B's 6 months. When they shared stock and gain, the first received \$228, and the second \$252. What was each man's amount of stock?

Let x represent A's capital, and y B's capital; then we have for the first Equation,

$$x+y=416.$$

Now, $228-x$ represents A's gain; and
$252-y$ represents B's gain.

And since the shares of profit are to each other in the compound ratio of *capital* and *time* (131), we have

$$228-x:252-y::9x:6y.$$

Converting this proportion into an Equation, uniting similar terms, and dividing both sides by 3,

$$456y+xy=756x.$$

From the first Equation,

$$y=416-x.$$

Substituting this value of y in the second Equation, and reducing,

$$x^2+796x=189696;$$

from which $x=\$192$; and $416-192=\$224$, is the value of y.

35. The sum of \$700 was divided among four persons, A, B, C, and D, whose shares were in geometrical progression; and the difference between the greatest and the least was to the difference between the two means as 37 to 12. What were the several shares?

Let x represent A's share, and y the ratio of the progression; then
xy, xy^2, and xy^3, respectively, represent B's, C's and D's shares.
By the conditions of the problem,

$$x+xy+xy^2+xy^3=700;$$

$$xy^3-x:xy^2-xy::37:12.$$

This proportion converted into an Equation becomes

$$12(xy^3-x)=37(xy^2-xy).$$

Dividing both sides of this Equation by $xy-x$,

$$12(y^2+y+1)=37y\,;$$

from which $y=\frac{4}{3}$.

Substituting this value of y for y in the first Equation, and adding similar terms,

$$\frac{175x}{27}=700\,;$$

from which $x=\$108$.

Hence, $xy=108\times\frac{4}{3}=\$144$; $xy^2=108\times\left(\frac{4}{3}\right)^2=\$192$; and $xy^3=108\times\left(\frac{4}{3}\right)^3=\$256$.

EXERCISES

On Affected Cubic and Biquadratic Equations.

1. Find the values of x in the Equation

$$x^3-6x^2+11x=6.$$

The divisors of the known term 6 are 1, 2, 3, 6; and it will be found that the Equation

$$x^3-6x^2+11x-6=0$$

is divisible by $x-1$, $x-2$, and $x-3$; hence 1, 2, and 3, are the roots of the given Equation, (254).

Or, having one of the roots as 1, the other two may be found from the quadratic Equation

$$x^2-5x+6=0,$$

which results from dividing the second Equation by $x-1$.

2. Find the values of x in the Equation

$$x^3-9x^2+26x=24.$$

The divisors of the absolute term 24, are 1, 2, 3, 4, 6, 12, and 24; and we shall find that the Equation

$$x^3-9x^2+26x-24=0,$$

is divisible by $x-2$, $x-3$, and $x-4$; hence 2, 3, and 4, are the roots of the given Equation.

3. Find the values of x in the Equation

$$x^3-3x^2-6x=-8.$$

The divisors of 8 are 1, 2, 4, and 8; and the Equation

$$x^3-3x^2-6x+8=0,$$

is divisible by $x-1$, $x+2$, and $x-4$; then 1, -2, and 4, are the required values of x.

4. Find the values of x in the Equation

$$2x^3-6x^2-8x=-24.$$

Dividing both sides of this Equation by 2, we have

$$x^3-3x^2-4x=-12.$$

The divisors of 12, are 1, 2, 3, 4, 6, and 12; and the Equation

$$x^3-3x^2-4x+12=0,$$

is divisible by $x-2$, $x+2$, and $x-3$; hence 2, -2, and 3, are the roots of the given Equation.

5, Find the values of x in the Equation

$$x^4+2x^3-13x^2-14x+24=0.$$

The divisors of the absolute term 24, are 1, 2, 3, 4, 6, 12, and 24; and the Equation is divisible by $x-1$, $x+2$, $x-3$, and $x+4$; then 1, -2, 3, and -4, are the required values of x.

Or, having found one of the roots, as 1, the other three may be found from the *cubic* Equation

$$x^3+3x^2-10x-24=0,$$

which results from dividing the given Equation by $x-1$.

6. Find an approximate value of x in the Equation

$$x^3+10x^2+5x=260.$$

First, It will be found that x is more than 4, and less than 5. Substituting these numbers for x, we have

64	x^3	125
160	$10x^2$	250
20	$5x$	25
244		400

Then, $400-244 : 5-4 :: 260-244$: the *correction* .1.

This correction added to the *less* assumed number, gives 4.1 for an approximate value of x.

Secondly, By substituting 4.1 and 4.2 for x, we have

68.921	x^3	74.088
168.10	$10x^2$	176.40
20.5	$5x$	21.00
257.521		271.488.

Then, $271.488-257.521 : 4.2-4.1 :: 260-257.521$: the *correction* .017.

Adding this correction to the less assumed number 4.1, we have 4.117 for a *nearer* value of x.

7. Find an approximate value of x in the Equation

$$x^3-15x^2+63x=50.$$

First, Substituting 1 and 1.5 for x,

1	x^3	3.375
-15	$-15x^2$	-33.75
63	$63x$	94.5
49		64.125.

Then, $64.125-49 : 1.5-1 :: 50-49$: the correction .03.

Hence we have $1+.03=1.03$, for an approximate value of x.

Secondly, Substituting 1.03 and 1.02 for x,

1.092727	x^3	1.061208
-15.9135	$-15x^2$	-15.606
64.89	$63x$	64.26
50.069227		49.715208.

Then, $50.069227-49.715208 : 1.03-1.02 :: 50-49.715208$: the correction .00804.

Hence $x=1.02+.00804=1.02804'$.

8. Find an approximate value of x in the Equation

$$x^3-17x^2+54x=350.$$

First, Substituting 14 and 15 for x,

2744	x^3	3375
−3332	$-17x^2$	−3825
756	$54x$	810
168		360.

Then, $360-168 : 15-14 :: 360-350$: the correction .05.

Subtracting this correction from the greater assumed member, we have 14.95 for an approximate value of x.

Secondly, Substituting 14.95 and 14.96 for x, we have

3341.362375	x^3	3348.071936
−3799.5425	$-17x^2$	−3804.6272
807.30	$54x$	807.84
349.119875		351.284736.

Then, $351.284736-349.119875 : 14.96-14.95 :: 350-349.119875$: the correction .004065.

Adding this correction to the less assumed number 14.95, we find 14.954065 for a *very near* value of x.

9. Find an approximate value of x in the Equation

$$x^4-3x^2-75x=10000.$$

First, Substituting 10 and 11 for x, we have

10000	x^4	14641
−300	$-3x^2$	−363
−750	$-75x$	−825
8950		13453.

Then, $13453-8950 : 11-10 :: 10000-8950$: the correction .23.
Hence 10.23 is the first approximation to the value of x.

Secondly, Substituting 10.2 and 10.3 for x,

10824.3216	x^4	11255.0881
−312.12	$-3x^2$	−318.27
−765	$-75x$	−772.5
9747.2016		10164.3181.

Then, 10164.3181−9747.2016 : 10.3−10.2 : : 10164.3181−10000 : the correction .039393.

Hence 10.3−.039393=10.260607, is a nearer value of x.

10. Find an approximate value of x in the Equation

$$2x^4-16x^3+40x^2-30x+1=0.$$

First, Substituting 1 and 2 for x,

2	$2x^4$	32
−16	$-16x^3$	−128
40	$40x^2$	160
−30	$-30x$	−60
1	1	1
−3		5.

Then, 5−(−3) : 2−1 : : 0−(−3) : the correction .3.
Therefore, 1+.3=1.3, is an approximate value of x.

Secondly, Substituting 1.3 and 1.2 for x,

5.7122	$2x^4$	4.1472
−35.152	$-16x^3$	−27.648
67.6	$40x^2$	57.6
−39	$-30x$	−36
1	1	1
.1602		−.9008.

Then, .1602−(−9008) : 1.3−1.2 : : 0−(−9008) : the correction .084.
Hence 1.2+.084=1.284, is a nearer value of x.

11. Find an approximate value of x in the Equation

$$\left(\frac{1}{5}x^2-15\right)^2+x\sqrt{x}=90.$$

Substituting 10 and 11 for x,

25	$\left(\frac{1}{5}x^2-15\right)^2$	84.64
31.622	$x\sqrt{x}$	36.482
56.622		121.122.

Then, 121.122−56.622 : 11−10 : : 121.122−90 : the correction .48.
Therefore 11−.48=10.52, is an approximate value of x.

PROBLEMS IN INEQUATIONS.

1. Find a number such, that when multiplied by 5, the product shall be less than 40, and when 5 is added to 3 times the square of the number, the sum shall be greater than 80.

Let x represent the number; then we shall have

$$5x<40, \text{ and } 3x^2+5>80.$$

Dividing the first Inequation by the coefficient of x, we find $x<8$. Transposing 5 in the second Inequation, and adding similar terms,

$$3x^2>75\,; \text{ hence } x^2>25, \text{ and } x>5.$$

The required number is therefore *indeterminate within the limits* 5 and 8; that is, it is either of the numbers 5, 6, or 7, plus any proper fraction.

2. What number is that whose third part diminished by 3, is greater than 20, and whose fourth part increased by 4, is less than 30?

Let x represent the number; then we have

$$\frac{x}{3}-3>20, \text{ and } \frac{x}{4}+4<30\,;$$

from which we shall find $x>69$, and <104.

3. Find a number whose square diminished by 10 is less than 90, and whose square root increased by 2 is greater than 5.

Let x represent the number; then

$$x^2-10<90\,; \text{ and } \sqrt{x}+2>5.$$

These Inequations will give $x<10$, and >9; hence the number is 9, plus any proper fraction.

4. Find a number such, that if it be multiplied by 2, 3, and 4, successively, the sum of the products shall be greater than 100, and if it be divided by the same numbers, the sum of the quotients shall be less than 30.

Let x represent the number; then

$$2x+3x+4x>100\,; \text{ and } \frac{x}{2}+\frac{x}{3}+\frac{x}{4}<30\,;$$

from which we shall find $x>11\frac{1}{9}$, and $<27\frac{9}{13}$.

5. A farmer sold a number of cattle. If 6 be subtracted from 3 times the number, the remainder will be less than the number increased by 48; and if 6 be added to 4 times the number, the sum will be greater than 27 increased by 3 times the number. What was the number of cattle?

Let x represent the number; then

$$3x-6<x+48;\ \text{and}\ 4x+6>27+3x;$$

from which $x<27$ and >21; hence the number of cattle was 22, 23, 24, 25, or 26.

6. Find a number whose square added to 4 times the number itself shall be more than 45, but whose square diminished by 10 times the number shall be less than 75.

Let x represent the number; then

$$4x+x^2>45;\ \text{and}\ x^2-10x<75;$$

Completing the square in the first member of each of these Inequations, we have

$$x^2+4x+4>49;\ \text{and}\ x^2-10x+25<100.$$

Extracting the square root of each member,

$$x+2>7;\ \text{and}\ x-5<10.$$

from which $x>5$ and <15.

7. Five times the number of miles between two places increased by 10, is less than 100; and 8 times the number diminished by 5, is greater than the number increased by 100. Required the number of miles between the two places.

Let x represent the number of miles; then

$$5x+10<100;\ \text{and}\ 8x-5>x+100.$$

These Inequations will give $x<18$ and >15; therefore the number of miles was 16, or 17, or either 15, 16, or 17, plus any proper fraction.

MISCELLANEOUS PROBLEMS.

1. A, B and C together have \$2000. B has \$100 less than twice as much as A, and C \$400 less than twice as much as the other two together. What sum has each?

Let x represent A's sum of money; then
$2x-100$ represents B's sum;
$2(3x-100)-400$ represents C's sum; and
$9x-700$ represents the whole sum.
The Equation will then be

$$9x-700=2000.$$

From this Equation we shall find $x=300$; then $2\times300-100=$ \$500, is B's sum, and $2000-300-500=$ \$1200, is C's sum.

2. A gentleman has three plantations. The first contains 250 acres, the second as much as the first and $\frac{1}{6}$ of the third, and the third as much as the first and second. What is the whole number of acres?

Let x represent the number of acres in the third plantation; then
$250+\frac{x}{6}$ represents the number in the second.
Then, since the third contains as much as the first and second,

$$x=500+\frac{x}{6};$$

from which $x=600$ acres; hence $250+\frac{600}{6}=350$ acres, the number in the second; and the first contains 250 acres. Then, $600+350+250=1200$ is the whole number of acres.

3. A company of workmen had been employed on a piece of work for 24 days, and had half finished it, when, by calling in the assistance of 16 more men, the remaining half was completed in 16 days. What was the original number of men?

Let x represent the original number of men; then
$x+16$ represents the number that finished the last half.
Since the time of performing each half of the work was *inversely* as the number of men employed on it,

$$x:x+16::16:24;$$

from which $x=32$

4. A's money was equal to $\frac{3}{4}$ of B's. A paid away \$50 less than $\frac{2}{3}$ of his, and B \$50 more than $\frac{3}{4}$ of his, when it was found that the latter had remaining only $\frac{1}{3}$ as much as the former. What sum had each at first?

Let x represent B's sum of money; then

$\frac{3x}{4}$ represents A's sum of money;

$\frac{2}{3}$ of $\frac{3x}{4}-50$, that is, $\frac{x}{2}-50$, is what A paid away;

$\frac{3x}{4}+50$, is what B paid away;

$\frac{x}{2}+50$ is what A had remaining; and

$\frac{x}{4}-50$ is what B had remaining.

By the conditions of the problem,

$$\frac{x}{4}-50-=\frac{1}{3}\left(\frac{x}{2}+50\right);$$

from which $x=\$400$, B's sum; hence $\frac{3}{4}$ of $400=\$300$, is A's sum.

5. A person wishing to enclose a piece of ground with palisades, found that if he set them one foot asunder, he would not have enough by 150; but if he set them one yard asunder, he would have too many by 70. What was the number of his palisades?

Let x represent the number; then since, if he set them one foot apart he would need 150,

$x+150$ represents the number of feet around the enclosure.

Since, if he set them one yard, or *three feet*, apart, he would have too many by 70,

$3x-70\times3$, or $3x-210$, also represents the number of feet around the enclosure.

We shall then have the Equation

$$x+150=3x-210;$$

which will give $x=180$ palisades.

6. From two tracts of land of equal size, were sold quantities in the proportion of 3 to 5. If 150 acres less had been sold from the one which is now the smaller of the two, only $\frac{2}{3}$ as much would have been taken from it as from the other. How many acres were sold from each?

Let x represent the number of acres sold from the first; then

$\frac{5x}{3}$ represents the number of acres sold from the second.

Since the second is now the smaller of the two, by the conditions of the problem,

$$\frac{5x}{3}-150=\frac{2x}{3};$$

from which we shall find $x=150$ acres; hence $\frac{5}{3}$ of $150=250$, the number sold from the second tract.

7. A and B had adjoining farms, which in quantity were in the ratio of 4 to 5. A sold to B 50 acres, and afterwards purchased from B one-third of his entire tract, when it was found that the original ratio of their quantities of land had been *reversed*. How many acres had each at first?

Let x represent the number of acres B had; then

$\frac{4x}{5}$ represents the number A had;

$\frac{4x}{5}-50$ represents the number A had after selling 50 acres;

$x+50$ represents the number B had after buying 50 acres;

$\left(\frac{4x}{5}-50\right)+\frac{1}{3}(x+50)$, or $\frac{17x}{15}-\frac{100}{3}$, is the number A had after buying from B; and

$\frac{2}{3}(x+50)$ represents the number B had after selling to A.

Reversing the original ratio of the quantities of land,

$$\frac{17x}{15}-\frac{100}{3}:\frac{2}{3}(x+50)::5:4.$$

Developing the second term of this proportion, and multiplying together extremes and means.

$$\frac{68x}{15}-\frac{400}{3}=\frac{10x}{3}+\frac{500}{3};$$

which will give $x=250$ acres, B's tract; hence $\frac{4}{5}$ of $250=200$ acres, A's tract.

8. A waterman can row down the middle of the stream, on a certain river, 5 miles in $\frac{3}{4}$ of an hour; but it takes him $1\frac{1}{2}$ hours to return, though he keeps along shore where the current is but half as strong as in the middle. What is the velocity of the middle of the stream?

Let x represent the velocity per hour; then since

$5 \div \frac{3}{4}$, or $\frac{20}{3}$, is the number of miles he can row down stream in one hour,

$\frac{20}{3} - x$ is the number he can row in one hour, *unassisted* by the stream.

As the current along shore is but half as strong as in the middle,

$\left(\frac{20}{3} - x - \frac{x}{2}\right)$ is the number of miles he travels per hour up stream; and

$\left(\frac{20}{3} - x - \frac{x}{2}\right) \times 1\frac{1}{2}$, or $10 - \frac{9x}{4}$, is the whole number of miles he travels up stream.

Since this distance is equal to the distance he travels down stream in $\frac{3}{4}$ of an hour, we have

$$10 - \frac{9x}{4} = 5;$$

from which we shall find $x = 2\frac{2}{9}$ miles per hour.

9. A farmer has three flocks of sheep, whose numbers are in the proportion of 2, 3, and 5. If he sell 20 from each flock, the whole number will be diminished in the proportion of 4 to 3. How many sheep has he in each flock?

Let x represent the number in the first flock; then

$\frac{3x}{2}$ represents the number in the second;

$\frac{5x}{2}$ represents the number in the third.

By the conditions of the problem,

$$(x-20) + \left(\frac{3x}{2} - 20\right) + \left(\frac{5x}{2} - 0\right) = \frac{3}{4}\left(x + \frac{3x}{2} + \frac{5x}{2}\right)$$

This Equation may easily be reduced to

$$5x - 60 = \frac{30x}{8};$$

which gives $x = 48$; hence $\frac{3}{2}$ of $48 = 72$, is the number in the second flock; and $\frac{5}{2}$ of $48 = 120$, is the number in the third flock.

10. The sum of $1170 is to be divided between three persons, A, B, and C, in proportion to their ages. Now, A's age is to B's as 1 to $1\frac{1}{3}$, and to C's as 1 to 2. What are the respective shares?

Let x represent A's share; then
$1\frac{1}{3}x$ represents B's share; and
$2x$ represents C's share.
The Equation will then be

$$x+1\tfrac{1}{3}x+2x=1170;$$

from which we shall find $x=\$270$, A's share; hence $270\times 1\frac{1}{3}=\$360$, is B's share; and $270\times 2=\$540$, is C's share.

11. A hare is 50 leaps before a greyhound, and takes 4 leaps to the greyhound's 3; but 2 of the greyhound's leaps are as much as 3 of the hare's. How many leaps must the greyhound take to catch the hare?

Let x represent the number of the greyhound's leaps; then
$\frac{4x}{3}$ represents the number the hare makes in the same time; and
$\frac{4x}{3}+50$ is the whole number of the hare's leaps.

Since 2 of the greyhound's leaps are as much as 3 of the hare's, each one of the greyhound's leaps will be equal to $\frac{3}{2}$ of *one of the hare's;* and the whole number of leaps will be *inversely* as the length of each leap; that is

$$x:\frac{4x}{3}+50::1:\frac{3}{2};$$

from which x will be found $=300$.

12. A vintner has two casks of wine, the contents of which are in the proportion of 5 to 6, and if $\frac{1}{6}$ of the quantity in the second were to be drawn off, the contents of the two casks would be equal. How many gallons are there in each?

Let x represent the number of gallons in the second; then
$\frac{5x}{6}$ represents the number in the first; and
$\frac{5x}{6}$ represents the quantity remaining in the second after draining off $\frac{1}{6}$.

By the conditions of the problem, we shall have

$$\frac{5x}{6}=\frac{5x}{6}.$$

This is an *identical* Equation; the problem is therefore *indeterminate*, (271).

13. A person looking at his watch, and being asked what o'clock it was, replied that it was between *eight* and *nine*, and that the hour and minute hands were exactly together. What was the time?

Let x represent the number of *minute* spaces moved over by the hour hand since 8 o'clock; then

$12x$ represents the number of minute spaces moved over by the minute hand in the same time.

But since there are 40 minute spaces between *twelve* and *eight*, it is evident that the minute hand has passed over $40+x$ spaces before overtaking the hour hand; hence we have the Equation

$$12x=40+x;$$

from which $x=3\frac{7}{11}$ minute spaces; hence $3\frac{7}{11}\times12=43\frac{7}{11}$ minute spaces, is the distance passed over by the minute hand since 8 o'clock. The time was therefore $43\frac{7}{11}$ minutes, or 43 minutes $38\frac{2}{11}$ seconds past *eight*.

14. A criminal having escaped from prison, traveled 10 hours before his escape was known. He was then pursued, and gained upon 3 miles an hour. When his pursuers had been 8 hours on the way, they met an express going at the same rate as themselves, who had met the criminal 2 hours and 24 minutes before. In what time from the commencement of the pursuit will the criminal be overtaken?

Let x represent the number of hours, and y the number of miles the criminal went per hour; then

$y+3$ represents the number of miles his pursuers went per hour;

$(10+x)y$ represents the number of miles the criminal traveled from the time of his escape; and

$(y+3)x$ represents the number of miles his pursuers traveled from the commencement of the pursuit.

We shall then have the following Equation:

$$(10+x)y=(y+3)x.$$

$(y+3)8$ represents the distance the pursuers had gone when they met the express; and

$18y$ represents the distance the criminal had gone when his pursuers met the express.

Now, the distance the criminal was in advance of his pursuers at this time, was equal to the sum of the distances the express and criminal had gone since their time of meeting; therefore, 2 hours and 24 minutes being equal to $2\frac{2}{5}$ hours,

$(y+3)2\frac{2}{5}+y\times2\frac{2}{5}$, or $\dfrac{24y+36}{5}$, represents the distance that the criminal was *in advance* of his pursuers when they met the express.

The second Equation will then be

$$18y=(y+3)8+\frac{24y+36}{5};$$

from which y is easily found $=6$; and substituting this value in the first Equation, we shall find $x=20$ hours.

15. A regiment of militia containing 875 men, is to be raised from three counties, A, B, and C. The quotas of A and B are in the proportion of 2 to 3, and of B and C in the proportion of 4 to 5. What is the number to be raised by each?

Let x represent the number to be raised by A; then

$\frac{3x}{2}$ represents the number to be raised by B; and

$\frac{5}{4}$ of $\frac{3x}{2}$, or $\frac{15x}{8}$, represents the number to be raised by C; then

$$x+\frac{3x}{2}+\frac{15x}{8}=875;$$

from which $x=200$ men; hence $\frac{3}{2}$ of $200=300$, is B's quota; and $\frac{15}{8}$ of $200=375$ men, is C's quota.

16. If 19 pounds of gold, in air, weighs 18 pounds in water; 10 pounds of silver, in air, weighs 9 in water; and a mass of 106 pounds, composed of gold and silver, weighs 99 pounds in water; what are the respective quantities of gold and silver in the mass?

Let x represent the number of pounds of gold, and y the number of pounds of silver; then

$$x+y=106.$$

Now, it is evident that each pound of gold weighs $\frac{18}{19}$ as much in water as in air, each pound of silver $\frac{9}{10}$ as much, and each pound of the whole mass $\frac{99}{106}$ as much; hence

$\frac{18x}{19}$ represents the weight of the whole quantity of gold, in water;

$\frac{9y}{10}$ represents the weight of the whole quantity of silver in water; and

$\frac{99(x+y)}{106}$ represents the weight of the whole mass of gold and silver, in water.

We shall then have the Equation

$$\frac{18x}{19}+\frac{9y}{10}=\frac{99(x+y)}{106};$$

from which and the first Equation, $x=76$, and $y=30$ pounds.

17. A farmer having mixed a certain quantity of corn and oats, found that if he had taken 6 bushels more of each, there would have been 7 bushels of corn to 6 of oats; but if he had taken 6 bushels less of each, there would have been 6 bushels of corn to 5 of oats. How many bushels of each were mixed?

Let x represent the number of bushels of corn, and y the number of oats; then the Equations will evidently be

$$x+6=\frac{7}{6}(y+6);$$

$$x-6=\frac{6}{5}(y-6);$$

from which we shall find $x=78$, and $y=76$ bushels.

18. Two persons, A and B, can perform a piece of work in 16 days. They work together for 4 days, when A being called off, B is left to finish it, which he does in 36 days more. In what time could each do it separately?

Let x represent the number of days in which A could do the work, and y the number in which B could do it; then

$\frac{1}{x}$ represents the part A could do in 1 day;

$\frac{1}{y}$ represents the part B could do in 1 day; also

$\frac{1}{16}$ is the part both could do in one day.

Therefore the first Equation will be

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{16}.$$

Since A and B together did $\frac{4}{16}$, or $\frac{1}{4}$ of the work in 4 days, $\frac{3}{4}$ of the work was left to be done by B, which, by the problem, he finished in 36 days. Therefore 36 days is $\frac{3}{4}$ of the time in which B could do the work alone; hence the second Equation will be

$$\frac{3y}{4}=36;$$

from which $y=48$ days; and x is now easily found $=24$ [illegible]ys.

19. A merchant has two casks containing unequal quantities of wine. Wishing to have the same quantity in each, he pours from the first into the second as much as the second contained at first; then he pours from the second into the first as much as was left in the first; and then again from the first into the second as much as was left in the second, when there are found to be 16 gallons in each cask. How many gallons did each cask contain at first?

Let x represent the number of gallons in the first, and y the number in the second cask; then, after pouring from the first into the second.

$x-y$ is the quantity in the first; and
$2y$ is the quantity in the second.

After pouring from the second into the first,

$(x-y)+(x-y)$, or $2x-2y$, is the quantity in the first;
$2y-(x-y)$, or $3y-x$, is the quantity in the second.

After again pouring from the first into the second,

$(2x-2y)-(3y-x)$, or $3x-5y$, is the quantity in the first,
$(3y-x)+(3y-x)$, or $6y-2x$, is the quantity in the second.

By the conditions of the problem, the Equation will now be

$$3x-5y=16;$$
$$6y-2x=16;$$

from which $x=22$, and $y=10$ gallons.

20. A fisherman being asked how many fish he had caught, replied, "If 5 be added to one-third of the number that I caught yesterday, it will make half the number I have caught to-day; or, if 5 be subtracted from three times this half, it will leave the number I caught yesterday." How many were caught each day?

Let x represent the number caught yesterday; then

$\frac{x}{3}+5$ represents half the number caught to-day.

By the conditions of the problem, the Equation will now be

$$3\left(\frac{x}{3}+5\right)-5=x.$$

This Equation can be reduced to

$$x-x=10; \text{ or } 0x=10.$$

$$\text{Hence } x=\frac{10}{0}=\infty, (50).$$

The problem is therefore impossible, (272).

21. A laborer engaged for n days, on condition that he should receive p pence for each day that he worked, and forfeit q pence for each day that he idled. At the end of the time he received s pence. How many days did he work? and how many was he idle?

Let x represent the number of days he worked; then
$n-x$ represents the number of days he idled;
px represents the number of pence he earned;
$(n-x)q$ represents the number of pence he forfeited.
We shall therefore have

$$px-(n-x)q=s;$$

which will give $x=\frac{nq+s}{p+q}$; hence $n-\frac{nq+s}{p+q}=\frac{np-s}{p+q}$, was the number of days he idled.

22. A, B, and C, engage in a joint speculation. A invests \$2000 for 5 months, B \$2400 for 4 months, and C \$1600 for 7 months. The profits amount to \$4620. What is each man's share of profit?

Let x represent A's share of profit; then

$$2000\times5:2400\times4::x:\frac{24x}{25}, \text{ B's share, (150).}$$

$$2000\times5:1600\times7::x:\frac{28x}{25}, \text{ C's share}$$

The Equation will then be

$$x+\frac{24x}{25}+\frac{28x}{25}=4620;$$

from which we shall find $x=\$1500$, A's share; hence $\frac{24}{25}$ of $1500=$ \$1440, is B's share, and $\frac{28}{25}$ of $1500=\$1680$, is C's share of profit.

23. A farmer wishes to mix rye worth 40 cents a bushel, and oats worth $26\frac{2}{3}$ cents a bushel, in such quantities as to produce 100 bushels which shall be worth 30 cents a bushel. What quantity of each must be taken?

Let x represent the number of bushels of rye, and y the number of bushels of oats; then

$$x+y=100.$$

Now, $40x$ represents the value of the rye, in cents;
$26\frac{2}{3}\,y$ represents the value of the oats, in cents; and
3000 cents is the value of the mixture.

The second Equation will therefore be

$$40x+26\tfrac{2}{3}\,y=3000\,;$$

from which and the first, $x=25$, and $y=75$ bushels.

24. Three persons engage in a joint mercantile adventure, in which the first has the capital a for the time b, the second the capital c for the time d, and the third the capital e for the time f. Their profits amount to s. What is each partner's share of profit?

Let x represent A's profit; then, since the respective shares of profit are in the compound ratio of capital and time,

$$ab:cd::x:\frac{cdx}{ab},\text{ B's profit, (150);}$$

$$ab:ef::x:\frac{efx}{ab},\text{ C's profit.}$$

We shall now have the following Equation:

$$x+\frac{cdx}{ab}+\frac{efx}{ab}=s.$$

This Equation will give $x=\dfrac{abs}{ab+cd+ef}$; then multiplying this value of x by $\dfrac{cd}{ab}$, we find $\dfrac{cds}{ab+cd+ef}$ for B's profit, and multiplying the same value by $\dfrac{ef}{ab}$, we find $\dfrac{efs}{ab+cd+ef}$ for C's profit.

25. A church which cost \$40000 is insured, annually, at $1\frac{1}{2}$ per cent, for such an amount, that, in case of its being destroyed by fire, the Insurance Company shall be liable for the cost of the edifice, and the premium of Insurance. What is the sum insured?

Let x represent the sum insured; then

$$100:100-1\tfrac{1}{2}:x:40000\,;$$

which, converted into an Equation, will give $x=\$40609.13'$.

26. Four towns are situated in the order of the first four letters of the alphabet. The distance from A to D is 34 miles; the distance from A to B is to the distance from C to D as 2 to 3; and $\frac{1}{4}$ of the distance from A to B added to half the distance from C to D, is three times the distance from B to C. What are the respective distances?

Let x represent the distance from A to B; then

$\frac{3x}{2}$ represents the distance from C to D; and

$\frac{1}{3}\left(\frac{x}{4}+\frac{1}{2} \text{ of } \frac{3x}{2}\right)$, or $\frac{x}{3}$, represents the distance from B to C

Since the distance from A to D is 34 miles,

$$x+\frac{3x}{2}+\frac{x}{3}=34;$$

from which $x=12$ miles; hence $\frac{3}{2}$ of $12=18$ miles, is the distance from C to D, and $\frac{1}{3}$ of $12=4$ miles is the distance from B to C.

27. A commission merchant receives the sum of s dollars to invest in merchandise—himself to retain a commission of r per cent on the amount of the purchase. What is the sum to be invested?

Let x represent the sum to be invested; then

$$100:100+r::x:s;$$

from which $x=\frac{100s}{100+r}$.

28. A sum of money was equally divided among a number of persons, by first giving to A \$100 and $\frac{1}{6}$ of the remainder, then to B \$200 and $\frac{1}{6}$ of the remainder, then to C \$300 and $\frac{1}{6}$ of the remainder, and so on. What was the sum divided? and the number of persons?

Let x represent the number of dollars each person received, and y the number of persons; then

xy represents the whole sum divided.

Since the first received 100 dollars and $\frac{1}{6}$ of the remainder,

$$x=100+\frac{xy-100}{6}.$$

Now, $xy-\left(100+\frac{xy-100}{6}\right)$, or $\frac{5xy-500}{6}$, represents the remainder of the sum divided after A received his portion; and the second Equation will be

$$x=200+\frac{1}{6}\left(\frac{5xy-500}{6}-200.\right)$$

The two Equations may, respectively, be reduced to

$$(3) \qquad 6x-xy=500;$$

$$(4) \qquad 36x-5xy=5500.$$

Multiplying Equation (3) by 6, and subtracting the product from the fourth Equation, we find

$$xy=2500 \text{ dollars, the whole sum.}$$

Hence $x=\frac{2500}{y}$, and substituting this value of x in the third Equation, we have

$$\frac{15000}{y}-2500=500;$$

from which $y=5$ persons.

29. The sum of \$500 is to be applied in part towards the payment of a debt of \$900, now due, and in part to paying the interest, at 7 per cent, in advance, on the remainder of the debt, on which a credit of 12 months is to be allowed. What is the amount of payment that can be made on the debt?

Let x represent the payment; then

$(900-x)\frac{7}{100}$ represents the interest on the remainder.

Hence we shall have

$$x+(900-x)\frac{7}{100}=500;$$

whence $x=\$469.89'$.

30. A besieged garrison had such a quantity of bread as would, if distributed to each man at 10 ounces a day, last 6 weeks; but having lost 1200 men in a sally, the governor was enabled to increase the allowance to 12 ounces per day. What was the original number of men?

Let x represent the original number of men; then

$10x$ represents the number of ounces of bread that would have been distributed to them, daily; and

$(x-1200)12$ represents the number that was distributed, daily, after 1200 men were lost.

The Equation will then evidently be

$$10x=(x-1200)12;$$

from which $x=7200$ men.

31. A is indebted to B the sum of s dollars, and is able to raise but a dollars. With this latter sum A proposes to pay a part of the debt, and the interest, at r per cent, in advance, on his note at n years, for the remainder. For what sum should the note be drawn?

Let x represent the sum for which the note should be drawn; then

$\frac{nrx}{100}$ represents the interest on the note for n years; and

$s-x$ represents the present payment.

Hence we shall have the following Equation

$$s-x+\frac{nrx}{100}=a;$$

from which $x=\frac{100(s-a)}{100-rn}$.

32. The crew of a ship consisted of her complement of sailors and a number of soldiers. Now, there were 22 sailors to every three guns, and 10 over. Also the whole number of men was 5 times the number of soldiers and guns together. But after an engagement, in which the slain were $\frac{1}{4}$ of the survivors, there wanted 5 of being 13 men to every 2 guns. Required the number of guns, soldiers, and sailors.

Let x represent the number of guns, y the number of soldiers, z the number of sailors, and w the number slain in the engagement; then

$y+z-w$ represents the number of survivors after the engagement.

Since there were 22 sailors to every 3 guns, there were $\frac{22}{3}$ to each gun, and $\frac{22x}{3}$ to x guns.

By the problem, the first Equation will therefore be

$$z=\frac{22x}{3}+10;$$

and the second will be

$$y+z=5y+5x.$$

After the engagement, the slain being $\frac{1}{4}$ of the survivors,

$$w=\frac{y+z-w}{4};$$

and as there now wanted 5 of being 13 men to every 2 guns,

$$y+z-w+5=\frac{13x}{2}.$$

From the third Equation we shall find $w=\frac{y+z}{5}$.

Substituting this value of w in the fourth Equation,

$$\frac{4y+4z}{5}+5=\frac{13x}{2};$$

from which and the first two Equations, we shall find $x=90$, $y=55$ and $z=670$.

33. Two sums of money amounting together to $600, were put at interest—the smaller at 2 per cent more than the other. The interest of the larger sum was afterwards increased, and that of the smaller diminished, 1 per cent. By this the interest of the whole was augmented one-twentieth. But if the interest of the greater sum had been so increased, without any diminution of the other, the interest of the whole would have been increased *one-tenth.* What were the two sums? and the two rates of interest?

Let x represent the larger sum, y the smaller, z the rate of interest of the larger, and w the rate of interest of the smaller; then

$$x+y=600;$$
$$w-z=2.$$

Now, $\frac{xz}{100}+\frac{yw}{100}$, or $\frac{xz+yw}{100}$, represents the interest of both sums, (174);

$\frac{x(z+1)}{100}+\frac{y(w-1)}{100}$, or $\frac{x(z+1)+y(w-1)}{100}$, represents the interest of the whole, after the first rate of interest was increased, and the second diminished, by 1 per cent.

By the problem, the third Equation will now be

$$(3)\quad \frac{x(z+1)+y(w-1)}{100}=\frac{xz+yw}{100}+\frac{1}{20}\times\frac{xz+yw}{100}.$$

But $\frac{x(z+1)}{100}+\frac{yw}{100}$, or $\frac{x(z+1)+yw}{100}$, represents the interest of the whole, after the first rate of interest was increased by 1 per cent, without any diminution of the second.

Hence the fourth Equation will be

$$(4)\quad \frac{x(z+1)+yw}{100}=\frac{xz+yw}{100}+\frac{1}{10}\times\frac{xz+yw}{100}.$$

The third and fourth Equations may respectively be reduced to

$$(5)\quad 20x=20y+xz+yw;$$
$$(6)\quad 10x=xz+yw.$$

Subtracting the sixth Equation from the fifth, and dividing by 10, we find

$$x=2y.$$

Substituting this value of x in the first Equation, we shall find $y=\$200$; hence $x=200\times 2=\$400$.

Substituting these values of x and y and the value of w from the second Equation, for x, y, and w, in the sixth Equation, we shall find $z=6$ per cent; then $w=6+2=8$ per cent.

34. Two persons purchase 300 acres of land at $2 per acre, each one paying $300; but the first takes the more fertile portion of the tract, at 25 cents above the mean price per acre, and the second the remainder at 25 cents below the mean price per acre. How many acres has each?

Let x represent the number the first has, and y the number the second has; then

$$x+y=300;$$

and as the mean price per acre is 2 dollars, and both together pay 600 dollars,

$$2x+2y=600.$$

Also, as the first pays 25 cents above the mean price per acre, he pays $2\frac{1}{4}$, or $\frac{9}{4}$ dollars per acre; and since the second pays 25 cents below the mean price, he pays $1\frac{3}{4}$, or $\frac{7}{4}$ dollars per acre; therefore

$$\frac{9x}{4}+\frac{7y}{4}=600.$$

The conditions of this problem thus gives rise to more independent Equations than there are unknown quantities to be determined; the problem is therefore *impossible*, (272).

35. The *area* of a field is 432 square rods, and the sum of its length and breadth is equal to twice their difference. Required the length and breadth of the field.

Let x represent the length; then

$\frac{432}{x}$ represents the breadth; and by the conditions of the problem,

$$x+\frac{432}{x}=2\left(x-\frac{432}{x}.\right)$$

Developing the second member, and clearing the Equation of fractions,

$$x^2+432=2x^2-864;$$

from which x will be found equal to 36 rods; hence $\frac{432}{36}=12$ rods, is the breadth.

36. A and B lay out some money on speculation. A disposes of his interest in the business for £11, and gains as much per cent as B lays out; B gains £36, and it appears that A gains 4 times as much per cent as B. What was the capital of each?

Let x represent A's capital, and y B's capital; then

$11-x$ represents A's gain;

$\frac{11-x}{x}$ represents his gain on one pound;

$\frac{(11-x)100}{x}$ represents his gain per cent.

Hence, since he gained as much per cent as B lays out,

$$\frac{(11-x)100}{x}=y.$$

$\frac{36}{y}$ represents B's gain on one pound; and

$\frac{3600}{y}$ represents his gain per cent.

The second Equation will then be

$$\frac{(11-x)100}{x}=4\times\frac{3600}{y}.$$

Clearing the first Equation of its fraction,

$$(11-x)100=xy.$$

Clearing the second of fractions,

$$(11-x)100y=14400x.$$

Multiplying the third Equation by y,

$$(11-x)100y=xy^2.$$

Subtracting this Equation from the one preceding it, transposing, and dividing both sides by x,

$$y^2=14400;\text{ or } y=£120.$$

Substituting this value of y in the third Equation, we shall find $x=£5$.

37. A garden which is 12 rods in length, and 8 rods in breadth, is surrounded by a walk whose area is equal to $\frac{1}{6}$ of the area of the garden itself. Required the breadth of the walk.

Let x represent the breadth of the walk; then

$12+2x$ represents its length on each of two opposite sides of the garden; and

8 rods is its length on each of the other two opposite sides;

$(12+2x)2+16$, or $40+4x$, is the whole length of the walk;

$(40+4x)x$ represents the area of the walk.

Since the area of the garden is 12×8, or 96 square rods, by the conditions of the problem,

$$(40+4x)x=\frac{96}{6}=16.$$

From this Equation we shall find $x=-5+\sqrt{29}$.

38. A and B hired a pasture into which A put 4 horses, and B as many as cost him 18 shillings a week. Afterwards B put in two additional horses, and found that he must pay 20 shillings a week. At what rate was the pasture hired?

Let x represent the number of shillings A paid, per week, and y the number of horses B had in the pasture; then

$x+18$ represents the number of shillings for which the pasture was hired, per week.

$\frac{x}{4}$ is what A paid, per week, for each of his horses;

$\frac{18}{y}$ is what B paid, per week, for each of his horses,

The first Equation will therefore be

$$\frac{x}{4}=\frac{18}{y}.$$

Now, after B put in two additional horses, and paid 20*s*., or 2 shillings more than at first,

$x-2$ represents the number of shillings A paid per week;

$(x-2)+20$, or $x+18$, represents the number of shillings both paid for the pasture, per week;

$\frac{x-2}{4}$ is what A paid, per week, for each of his horses;

$\frac{20}{y+2}$ is what B paid, per week, for each of his horses.

The second Equation will then evidently be

$$\frac{x-2}{4}=\frac{20}{y+2}.$$

From the first Equation,

$$xy=72\text{; or } y=\frac{72}{x}.$$

From the second Equation,

$$xy+2x-2y=84.$$

Substituting the values of xy and y, as found above, in this Equation,

$$72+2x-\frac{144}{x}=84;$$

which will give $x=12$ shillings; hence $12+18=30$ shillings, is the sum for which the pasture was hired.

39. A gentleman bought a rectangular piece of ground, at \$10 for every rod in its *perimeter*. If the same area had been in the form of a square, and had been purchased in the same way, it would have cost \$20 less; and a square piece of the same perimeter would have contained $12\frac{1}{4}$ square rods more. What were the length and breadth of the lot?

Let x represent the length, and y the breadth of the lot; then

$2x+2y$ represents the perimeter of the lot;

$(2x+2y)10$ represents the number of dollars for which it was purchased.

xy represents the area of the lot;

$\sqrt{xy}$ represents one side of a square lot of ground of the same area as the former;

$4\sqrt{xy}$ represents its perimeter; and

$40\sqrt{xy}$ represents its cost if purchased in the same way as the former.

By the conditions of the problem, we shall then have

$$40\sqrt{xy}=(2x+2y)10-20.$$

Now, $\frac{2x+2y}{4}$ represents one side of a square piece of ground of the same perimeter as the first lot;

$\left(\frac{2x+2y}{4}\right)^2$ represents the area of such a piece of ground.

Hence we shall have for the second Equation,

$$\left(\frac{2x+2y}{4}\right)^2=xy+12\tfrac{1}{4}.$$

Squaring both sides of the first Equation, and dividing by 400, we have

$$4xy=(x+y)^2-2(x+y)+1\,;$$

which may easily be reduced to

$$(4) \qquad 2xy-x^2+2x-y^2+2y=1.$$

Developing the first member of the second Equation, clearing of fractions, &c.,

$$4x^2-8xy+4y^2=196.$$

Dividing both sides by 4, we find

$$x^2-2xy+y^2=49.$$

Extracting the square root of both sides,

$$x-y=7\,;\ \text{or } x=7+y.$$

Substituting this value of x in the fourth Equation, transposing, and adding similar terms,

$$4y=36\,;\ \text{or } y=9 \text{ rods.}$$

$$\text{Hence } x=7+9=16 \text{ rods.}$$

40. A person being asked the ages of himself and wife, replied, that the product of their ages added to the square of his age, would make 1560, but added to the square of hers would make 1144. What were their ages?

Let x represent the husband's age, and y the wife's age; then

$$x^2+xy=1560\,;$$
$$y^2+xy=1144.$$

Adding these Equations together,

$$x^2+2xy+y^2=2704.$$

Extracting the square root of both sides,

$$x+y=52\,;\ \text{or } x=52-y.$$

Substituting this value of x in the second Equation, and adding similar terms,

$$52y=1144\,;\ \text{or } y=22 \text{ years.}$$

$$\text{Hence } x=52-22=30 \text{ years.}$$

41. A and B purchased a farm containing 900 acres for which they paid \$900 each. On dividing the land it was agreed that A should have his choice of situation, and pay 45 cents per acre more than B. How many acres should each have taken? and at what price per acre?

Let x represent the number of acres in A's portion, and y the number of cents he should have paid per acre; then

$900-x$ represents the number of acres in B's portion;

$y-45$ represents the number of cents he should have paid per acre; and since 900 dollars is equal to 90000 cents, we shall have

$$xy=90000;$$

$$(900-x)(y-45)=90000.$$

Developing the first member of this Equation, and reducing,

$$900y-xy+45x=130500.$$

Substituting the values of xy and x from the first Equation, and reducing, we find

$$y+\frac{4500}{y}=245;$$

from which y will be found $=225$ cents$=\$2.25$.

We can now easily find $x=400$ acres; hence $900-400=500$ acres, was B's portion; and $225-45=180$ cents$=\$1.80$ was its price per acre.

42. A capital of $13000 was divided into two parts, which were put at interest in such a manner that the income was the same from each. If the first part had been at the same rate of interest as the second, it would have produced an income of $360; and if the second part had been at the same rate as the first, it would have produced an income of $490. What were the two rates of interest?

Let x represent the first of the parts into which the whole sum was divided, y its rate per cent of interest, and z the rate per cent of interest of the second part; then

$13000-x$ represents the second of the parts into which the whole sum was divided;

$\frac{xy}{100}$ represents the interest of the first part, (174);

$\frac{13000z-xz}{100}$ represents the interest of the second part.

The first Equation will then be

$$\frac{xy}{100}=\frac{13000z-xz}{100};$$

and by the conditions of the problem, the other two Equations will be

$$\frac{xz}{100}=360; \text{ and}$$

$$\frac{13000y-xy}{100}=490.$$

These three Equations may respectively be reduced to

$$(5)\qquad \begin{aligned} xy &= 13000z - xz\,; \\ xz &= 36000\,; \text{ and} \\ 13000y - xy &= 49000. \end{aligned}$$

Substituting the value of xz from the fifth Equation for xz in the fourth Equation,

$$xy = 13000z - 36000.$$

Substituting this value of xy for xy in the sixth Equation, transposing, and adding similar terms,

$$13000y - 13000z = 13000.$$

Dividing both sides by 13000, we find

$$y - z = 1\,; \text{ or } y = z + 1.$$

Substituting this value of y for y in the fourth Equation, and reducing,

$$2xz + x = 13000z.$$

Substituting for xz and x in this Equation their values from the fifth Equation, we have

$$72000 + \frac{36000}{z} = 13000z\,;$$

from which $z = 6$ per cent; and since $y = z + 1$, we have $y = 6 + 1 = 7$ per cent.

The first and second parts into which the whole sum was divided will now easily be found equal to \$6000 and \$7000, respectively.

43. A departs from London towards Lincoln at the same time at which B leaves Lincoln for London. When they met, A had traveled 20 miles more than B, having gone as far in $6\frac{2}{3}$ days as B had in all the time; and it appeared that B would not reach London under 15 days. What is the distance between the two places? and how far had each man traveled?

Let x represent the number of miles A had traveled, and y the number B had traveled; then

$$x = y + 20.$$

Since A went in $6\frac{2}{3}$ days as many miles as B had traveled when they met,

$$y \div 6\tfrac{2}{3}, \text{ or } \frac{3y}{20}, \text{ represents the distance A went per day;}$$

$$x \div \frac{3y}{20}, \text{ or } \frac{20x}{3y}, \text{ represents the number of days he had traveled}$$

when he met B.

Since, in 15 days, B will reach London, or travel the distance A had traveled when they met,

$x \div 15$, or $\frac{x}{15}$, represents the distance B went per day;

$y \div \frac{x}{15}$, or $\frac{15y}{x}$, represents the number of days he had traveled when he met A.

The second Equation will therefore be

$$\frac{20x}{3y} = \frac{15y}{x};$$

which may be reduced to

$$4x^2 = 9y^2; \text{ or } x = \frac{3y}{2}.$$

Substituting the value of x in the first Equation, we shall find $y = 40$ miles; hence $x = \frac{3}{2}$ of $40 = 60$ miles; and $40 + 60 = 100$ miles is the whole distance.

44. The product of the two dimensions of a rectangular piece of land, subtraced from the square of the greater dimension, leaves 300 square rods, and subtracted from the square of the less, leaves 200 square rods. What are the dimensions of the piece?

Let x represent the length, and y the breadth; then

$$x^2 - xy = 300;$$
$$y^2 - xy = 200.$$

Decomposing the first member of the first Equation,

$$x(x - y) = 300.$$

Changing the signs of all the terms in the second Equation, and decomposing the first member of the resulting Equation,

$$y(x - y) = -200.$$

Dividing this Equation by the one preceding it, and reducing the resulting Equation to its lowest terms,

$$\frac{y}{x} = -\frac{2}{3}; \text{ or } y = -\frac{2x}{3}.$$

Substituting this value of y in the second Equation, we shall find $x = \sqrt{180} = 6\sqrt{5}$; hence $y = -\frac{2}{3}$ of $6\sqrt{5} = -4\sqrt{5}$.

The problem is therefore *impossible*, (272); and the impossibility consists in supposing that the product of the length and breadth of the piece of land is less than the square of the breadth.

45. A person being asked the ages of his two children, replied, that the difference of their ages was 3 years, and the product multiplied by the sum of their ages was 308. What were their ages?

Let x represent the age of the younger; then
$x+3$ represents the age of the older;
x^2+3x represents the product of their ages;
$2x+3$ represents their sum; and we shall have

$$(x^2+3x)(2x+3)=308.$$

Multiplying together the two binomials in the first member,

$$2x^3+9x^2+9x=308.$$

It will be easily seen that the value of x in this Equation must be some one of the *smaller* divisors of the known term 308, and we shall find that the Equation, with 308 transposed to the first member, is divisible by $x-4$; then 4 years was the age of the younger child, (254); and $4+3=7$ years, was the age of the older.

46. A gentleman who had a square lot of ground, reservered 10 square rods out of it, and sold the remainder for \$432, which was as many dollars per square rod as there were rods in a side of the whole square. What was the length of its sides?

Let x represent the number of rods in the length of each side; then
x^2 represents its area; and after 10 square rods of it were reserved,
$\frac{432}{x^2-10}$ represents the price for which it was sold, per square rod.

Hence we shall have the Equation

$$\frac{432}{x^2-10}=x.$$

Clearing this Equation of its fraction,

$$432=x^3-10x.$$

This Equation with all of its terms transposed to one side is divisible by $x-8$; hence, 8 rods was the length of its sides.

47. A and B set out together from the same place, and travel in the same direction. A goes the first day 28 miles, the second 26, and so on, in arithmetical progression; while B goes uniformly 20 miles per day. In how many days will the two be together again?

Let x represent the number of days in which they will be together again; then

$28-2(x-1)$ represents the number of miles A goes on the last day, (176);

$\frac{28+28-2(x-1)}{2}x$, or $29x-x^2$, represents the whole distance he travels, (180); and

$20x$ represents the whole distance B travels.

The Equation will therefore be

$$29x-x^2=20x;$$

from which $x=9$ days.

48. A farmer wishes to build a crib whose capacity shall be 1620 cubic feet, and whose length, breadth, and height shall be in arithmetical progression decreasing by the common difference 3. What must be the dimensions of the crib?

Let x represent the number of feet in length; then

$x-3$ represents the number in breadth;

$x-6$ represents the number in height; and the Equation will be

$$x(x-3)(x-6)=1620.$$

Multiplying together the binomials in the first member, and transposing 1620, we have

$$x^3-9x^2+18x-1620=0.$$

This Equation is divisible by $x-15$; then 15 feet must be the length; $15-3=12$ feet, the breadth; and $15-6=9$ feet, the height.

49. One traveler sets out to go from A to B, at the same time at which another sets out from B to A. They both travel uniformly, and at such rates, that the former, 4 hours after their meeting, arrives at B, and the latter at A, in 9 hours after. In how many hours did each one perform the journey?

Let x represent the number of hours in which the first performed the journey, and y the number in which the second performed it.

Since the first arrives at B in 4 hours after their meeting, and the second at A in 9 hours after their meeting,

$x-4$ represents the number of hours the first had traveled at the time of meeting;

$y-9$ represents the number of hours the second had traveled at the time of meeting; hence

$$x-4=y-9.$$

Now, the first travels in 4 hours, the distance traveled by the second in $(y-9)$ hours; therefore the number of hours in which the first performed the whole journey is to the number of hours in which the second performed it as 4 is to $y-9$; that is

$$x : y :: 4 : y-9.$$

Reducing this proportion to an Equation,

$$xy-9x=4y.$$

From the first Equation,

$$x=y-5.$$

Substituting this value of x in the second Equation, and reducing,

$$y^2-18y=-45;$$

from which $y=15$ hours; and as $x=y-5$, we have $x=15-5=10$ hours.

50. A lady on being asked the ages of her three little boys, answered that they were in harmonical progression; and the sum of their ages was 22 years; and that if the ages of the two elder were each increased by $\frac{1}{3}$ of itself, the three would then be in geometrical progression. What were their respective ages?

Let x represent the age of the youngest, and y the age of the next older; then

$22-x-y$ represents the age of the oldest; and as the ages are in harmonical progression,

$$x : 22-x-y :: y-x : (22-x-y)-y, \text{ (182).}$$

Now, increasing the ages of the two elder, each by $\frac{1}{3}$ of itself,

$\frac{4y}{3}$ represents the age of the second;

$\frac{4}{3}(22-x-y)$ represents the age of the oldest; and as the ages are now in geometrical progression (185),

$$\frac{4y}{3} \div x = \frac{4}{3}(22-x-y) \div \frac{4y}{3}.$$

Reducing the proportion to an Equation, transposing, and adding similar terms,

$$y^2-2xy=2x^2-44x+22y.$$

Performing the division indicated in each member of the first Equation, and reducing,

$$4y^2+3xy=66x-3x^2.$$

Multiplying the second Equation by 3 and the third by 2, we have

$$3y^2-6xy=6x^2-132x+66y;$$
$$8y^2+6xy=132x-6x^2.$$

Adding these Equations together,

$$11y^2=66y; \text{ hence } y=6 \text{ years.}$$

We can now easily find $x=4$ years; and $22-4-6=12$ years, the age of the oldest.

51. A person wishes to construct two cubical reservoirs which shall differ in their linear dimensions by 4 feet, and which shall together contain 5824 cubic feet. What must be the dimensions of the two reservoirs?

Let x represent the number of feet in each dimension of the less reservoir; then

$x+4$ represents the number of feet in each dimension of the greater;

x^3 represents the number of cubic feet in the capacity of the less;

$(x+4)^3$ represents the number in the capacity of the greater.

By the problem, we shall now have

$$x^3+(x+4)^3=5824.$$

Developing the binomial in the first member, adding similar terms, and dividing by 2,

$$x^3+6x^2+24x=2880.$$

Using the lower divisors of the known term 2880, we shall find that the preceding Equation with all its terms transposed to one side is divisible by $x-12$; hence 12 feet must be the length of each dimension of the less reservoir; and $12+4=16$ feet must be the length of each dimension of the greater.

52. Two partners, A and B, divided their gain, which was \$60, when B's share was found to be \$20. A's capital was in trade 4 months: and if the number 50 be divided by A's capital, the quotient will be in the number of months that B's capital, which was \$100, continued in trade. What was A's capital? and the time B's was in trade?

Let x represent the number of dollars in A's capital; then

$\frac{50}{x}$ represents the number of months B's capital continued in trade; and as B's gain was \$20, A's gain was $60-20$, or 40 dollars.

As the respective shares of gain are in the compound ratio of capital and time (131), we shall have

$$4x : 100\times\frac{50}{x} :: 40 : 20;$$

which converted into an Equation will give $x=\$50$; hence $\frac{50}{50}=1$ month, was the time B's money continued in trade.

53. Let there be a square whose side is 110 inches; it is required to assign the length and breadth of a rectangle whose perimeter shall be greater than that of the square by 4 inches, but whose area shall be less than the area of the square by 4 square inches.

Let x represent the length, and y the breadth; then

$2x+2y$ represents the perimeter of the required rectangle; and

Since 110×4, or 440 square inches is the perimeter of the square, we shall have the Equation

$$2x+2y=440+4.$$

Now, 110×110, or 12100 square inches, is the area of the square, and

xy represents the area of the required rectangle.

The second Equation will then be

$$xy=12100-4.$$

From the second Equation,

$$x=\frac{12096}{y}.$$

Substituting this value of x in the first Equation, and reducing,

$$2y^2-444y=24192;$$

which will give $y=96$ inches; and x is now easily found $=126$ inches.

54. There is a number consisting of three digits which increase from left to right by the common difference 2; and the product of the three digits is 105. Required the number.

Let x represent the hundreds' figure; then

$x+2$ represents the tens' figure;

$x+4$ represents the units' figure; and we shall have

$$x(x+2)(x+4)=105; \text{ or}$$
$$x^3+6x^2+8x=105.$$

We shall find that this Equation with 105 transposed to the first member is divisible by $x-3$; hence 3 is the hundreds' figure, $3+2=5$ is the tens' figure, and $3+4=7$ is the units' figure; therefore the number is 357.

55. A person bought two pieces of cloth for $63. For the first piece he paid as many dollars per yard as there were yards in both pieces, and for the second as many dollars per yard as there were yards in the first more than in the second; also the first piece cost six times as much as the second. What was the number of yards in each piece?

Let x represent the number of yards in the first piece, and y the number in the second; then

$x(x+y)$ represents the number of dollars he paid for the first piece; and

$y(x-y)$ represents the number of dollars he paid for the second piece.

We shall now have the Equations

$$x(x+y)+y(x-y)=63;$$
$$x(x+y)=6y(x-y).$$

Substituting the value of $x(x+y)$ from the second Equation for $x(x+y)$ in the first, and adding similar terms,

$$7y(x-y)=63; \text{ hence}$$
$$y(x-y)=9.$$

From this Equation we shall easily find

$$x=\frac{9+y^2}{y}.$$

Substituting this value of x, and the value of $y(x-y)$ as found above, for these quantities, respectively, in the second Equation,

$$\frac{81+27y^2+2y^4}{y^2}=54;$$

which may be reduced to

$$2y^4-27y^2=-81:$$

from which y will be found $=3$ yards, (263); and x is now easily found $=6$ yards.

56. There is a number consisting of four digits which decrease from left to right by the common difference 2; and the product of the four digits is 945. Required the number.

Let x represent the thousands' figure; then
$x-2$ represents the hundreds' figure;
$x-4$ represents the tens' figure;
$x-6$ represents the units' figure.
The Equation is

$$x(x-2)(x-4)(x-6)=945; \text{ or}$$
$$x^4-12x^3+44x^2-48x-945=0;$$

which will be found to be divisible by $x-9$; hence 9 is the thousands' figure, $9-2=7$ is the hundreds', $9-4=5$ is the tens', and $9-6=3$ is the units' figure; and the number is 9753.

57. A gentleman purchased two square lots of ground for \$300; each of them cost as many cents per square rod as there were rods in a side of the other, and the greater contained 500 square rods more than the less. What was the cost of each lot?

Let x represent the number of rods in a side of the greater lot, and y the number in a side of the less; then
x^2 represents the area of the greater lot;
y^2 represents the area of the less;
x^2y represents the cost of the greater, in cents;
y^2x represents the cost of the less, in cents; then, 300 dollars being equal to 30000 cents, the first Equation will be

$$x^2y+y^2x=30000;$$

and, by the problem, the second Equation will be

$$x^2-y^2=500.$$

Decomposing the first member of each of these Equations,

$$(x+y)xy=30000;$$
$$(x+y)(x-y)=500.$$

Dividing the first of these Equations by the second, and reducing the result to its lowest terms,

$$\frac{xy}{x-y}=60; \text{ hence}$$

$$(6) \qquad xy=60x-60y.$$

Now, assuming $x=z+w$, and $y=z-w$, and substituting these values of x and y in the sixth and second Equations,

$$(7) \qquad z^2-w^2=120w;$$
$$4zw=500.$$

From the last Equation,

$$z = \frac{125}{w}.$$

Substituting this value of z in the seventh Equation,

$$\frac{(125)^2}{w^2} - w^2 = 120w; \text{ or}$$

$$w^4 + 120w^3 = 15625;$$

from which $w=5$, (268); hence $z = \frac{125}{5} = 25$; $x = 25+5 = 30$ rods; $y = 25-5 = 20$ rods.

Therefore the greater lot cost $(30)^2 \times 20 = 18000$ cents $= 180$ dollars; and the less cost $(20)^2 \times 30 = 12000$ cents $= 120$ dollars.

58. A merchant bought a number of bales of cloth. The number of pieces in each bale was 10 more than the number of bales, and the number of yards in each piece was 5 more than the number of pieces in each bale; and the whole quantity was 1500 yards. What was the number of bales?

Let x represent the number of bales; then
$x+10$ represents the number of pieces in each bale;
$x+15$ represents the number of yards in each piece.
The Equation will now be

$$x(x+10)(x+15) = 1500; \text{ or}$$

$$x^3 + 25x^2 + 150x - 1500 = 0.$$

Taking the smaller divisors of the known term, it will be found that the preceding Equation is divisible by $x-5$; hence the number of bales was 5.

59. A person dies, leaving children, and a fortune of $46800, which, by his will, is to be divided equally amongst them. Immediately after the death of the father, two of the children also die, in consequence of which each surviving one receives $1950 more than he was entitled to by the will. How many children did the father leave?

Let x represent the number of children the father left; then

$\frac{46800}{x}$ represents the number of dollars bequeathed to each;

$\frac{46800}{x-2}$ represents the number each received *after two had* died;

and, by the conditions of the problem,

$$\frac{46800}{x-2}=\frac{46800}{x}+1950;$$

which may be reduced to

$$1950x^2-3900x=93600;$$

from which x will be found equal to 8.

60. A coach set out from Cambridge for London with 4 more *outside* than *inside* passengers. Seven outside passengers went at 2 shillings less than 4 inside ones, and the fare of the whole amounted to £9. At the end of half the journey, 3 more outside and one more inside passenger were taken up, in consequence of which the fare of the whole was increased in the proportion of 19 to 15. Required the number of passengers at first, and the fare of each.

Let x represent the number of inside passengers, y the number of shillings each one of them paid, and z the number of shillings each outside passenger paid; then

$x+4$ represents the number of outside passengers;

$4y$ represents the fare of four inside passengers;

$7z$ represents the fare of 7 outside passengers; and the first Equation will be

$$7z=4y-2.$$

$xy+(x+4)z$ represents the fare of the whole; and since £9 is equal to 180 shillings, the second Equation will be

$$xy+(x+4)z=180.$$

$(x+1)$ represents the number of inside passengers after 1 more was taken up;

$(x+7)$ represents the number of outside passengers after 3 more were taken up;

$(x+1)y+(x+7)z$ represents the fare of the whole number now in the coach; and by the problem, we shall now have

$$(x+1)y+(x+7)z : 180 :: 19 : 15.$$

From the first Equation,

$$(3) \qquad 7z-4y=-2.$$

From the second Equation,

$$(4) \qquad xy+xz+4z=180.$$

Multiplying together the extremes and means of the proportion, and dividing the resulting Equation by 15, we have

$$(5) \qquad xy+y+xz+7z=228.$$

Subtracting the fourth Equation from the fifth,

$$y+3z=48;$$

from which and the third Equation, we shall easily find $y=18$ shillings, and $z=10$ shillings; and substituting the values of y and z in the second Equation, we shall find $x=5$, the number of inside passengers; hence $5+4=9$, was the number of outside passengers.

61. There is a fraction, which inverted and increased by $\frac{1}{3}$ will be greater than 2; but if its numerator be increased by 2, the value of the fraction will be greater than $\frac{1}{2}$. What is the fraction?

Let $\frac{1}{x}$ represent the fraction; then

$\frac{x}{1}$, or x, represents the fraction inverted; and by the conditions of the problem,

$$x+\frac{1}{3}>2, \text{ and } \frac{3}{x}>\frac{1}{2}.$$

From the first Inequation we find

$$x>\frac{5}{3};$$

and from the second, $x<6$.

Then since the greater the *divisor* the less the quotient, and conversely, (147); dividing 1 by each of these values of x, we have

$$\frac{1}{x}<\frac{3}{5}, \text{ and, } \frac{1}{x}>\frac{1}{6}.$$

The required fraction is therefore less than $\frac{3}{5}$ and greater than $\frac{1}{6}$.

62. In a purse which contains 24 coins of silver and copper, each silver coin is worth as many pence as there are copper coins, each copper coin is worth as many pence as there are silver coins, and the whole is worth 18 shillings. How many were there of each kind of coins?

Let x represent the number of silver, and y the number of copper coins; then

$$x+y=24.$$

Now, $xy+yx$, or $2xy$, represents the whole value of the silver and copper coins; and since this value is 18 shillings, or 216 pence, we shall have

$$2xy=216.$$

From the second Equation,

$$x=\frac{108}{y};$$

and substituting this value in the first Equation, we shall find $y=18$; hence $x=\frac{108}{18}=6$.

63. The square root of a certain number plus 4 is less than 9; and ten times the square root of the number minus 2, is greater than 8 times the square root of the number plus 4. What is the number?

Let x represent the number; then

$$\sqrt{x}+4<9;$$
$$10\sqrt{x}-2>8\sqrt{x}+4.$$

From the first Inequation,

$$\sqrt{x}<5;\text{ or } x<25.$$

From the second,

$$2\sqrt{x}>6;\text{ or } x>9.$$

Hence the required number is between 9 and 25.

64. A and B traveled on the same road, and at the same rate, from Huntingdon to London. At the 50th mile stone from London, A overtook a drove of geese, which were proceeding at the rate of 3 miles in 2 hours; and 2 hours afterwards met a wagon which was moving at the rate of 9 miles in 4 hours. B overtook the same drove of geese at the 45th mile stone, and met the same wagon 40 minutes before he came to 31st mile stone. Where was B when A reached London?

Let x represent the number of miles each traveled per hour; then since the geese were moving at the rate of $\frac{3}{2}$ miles per hour, and were overtaken by B at the 45th mile stone, or 5 miles nearer London than when A overtook them,

$5\div\frac{3}{2}$, or $\frac{10}{3}$, is the number of hours that A had passed the 50th mile stone, when B came to the 45th mile stone;

$\frac{10x}{3}$ represents the distance A was from the 50th mile stone when B reached the 45th mile stone;

$\frac{10x}{3}-5$ *represents the distance A was in advance of B*

Since A met the wagon 2 hours after passing the 50th mile stone,

$50-2x$ represents the distance of the wagon from London when it met A.

As B met the wagon 40 minutes or $\frac{2}{3}$ of an hour before coming to the 31st mile stone,

$31+\frac{2x}{3}$, or $\frac{93+2x}{3}$, represents the distance of the wagon from London when it met B;

$\frac{93+2x}{3}-(50-2x)$, or $\frac{8x-57}{3}$, represents the distance the wagon moved from the time it met A to the time it met B;

$\frac{8x-57}{3}\div\frac{9}{4}$, or $\frac{32x-228}{27}$, represents the number of hours the wagon was moving the above distance;

$\frac{32x-228}{27}\times x$, or $\frac{32x^2-228x}{27}$, represents the distance A had traveled since meeting the wagon, at the time B met it.

Then this distance plus the distance the wagon had moved from the time of meeting A to the time of meeting B is equal to the *distance A was in advance of B.*

Hence we shall have the Equation

$$\frac{32x^2-228x}{27}+\frac{8x-57}{3}=\frac{10x}{3}-5;$$

which may easily be reduced to

$$32x^2-246x=378.$$

This Equation will give $x=9$ miles, the distance each traveled per hour; and as $\frac{10x}{3}-5$ represents the distance A was in advance of B, we have $\frac{10}{3}$ of $9-5$, or 25 miles, for the distance between them when A reached London.